For example, $\frac{7}{-9}$ can be written as either $\frac{-7}{9}$ or as

$-\frac{7}{9}$ (either in the numerator or in front of the fraction, but not both) .

However, if a fraction has two negatives, then it should first be simplified to a positive fraction. For example, the fraction $\frac{-6}{-8}$ has two negatives, making the whole fraction positive, and we should first write it as $\frac{6}{8}$ before simplifying it to $\frac{3}{4}$.

Think About It 4 If a single fraction has three negative signs, such as $-\frac{-9}{-12}$, will the fraction simplify to a positive or negative value? Explain your answer.

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Example 7: Simplify the fraction completely.

a) $\frac{10}{-15}$ b) $\frac{-12}{-20}$

Procedure: First decide whether the fraction is positive or negative, then simplify.

Answer:

a) $\frac{10}{-15}$ A single negative. Write the negative in front of the fraction.

$= -\frac{10}{15}$ $\frac{10}{15}$ simplifies by a factor of 5 to $\frac{2}{3}$.

$= -\frac{2}{3}$

b) $\frac{-12}{-20}$ Two negatives means this fraction is positive.

$= \frac{12}{20}$ $\frac{12}{20}$ simplifies by a factor of 4 to $\frac{3}{5}$.

$= \frac{3}{5}$

You Try It 7 Simplify the fraction completely. Use Example 7 as a guide.

a) $\frac{-9}{-27}$ b) $-\frac{25}{-35}$ c) $\frac{-14}{42}$ d) $\frac{24}{-60}$

MULTIPLYING SIGNED FRACTIONS

When multiplying or dividing two fractions, there can be many negative signs within. Before starting the actual multiplication or division, it is best to determine whether the result will be positive or negative. An odd number of negatives gives a result that is negative, and an even number of negatives gives a result that is positive.

Example 8: Evaluate.

a) $\frac{-2}{3} \cdot \frac{5}{-4}$ b) $\frac{-4}{9} \cdot \frac{15}{2}$

Procedure: First decide whether the end result will be positive or negative. If the end result will be negative, be sure to show the negative sign at each step.

Answer: a) $\frac{-2}{3} \cdot \frac{5}{-4}$ Two negatives mean the end result is positive. To emphasize this, place a plus sign in front and eliminate all negative signs.

$= +\frac{2}{3} \cdot \frac{5}{4}$ Multiply and simplify. $\frac{2}{3} \cdot \frac{5}{4}$ simplifies by a factor of 2 to $\frac{1}{3} \cdot \frac{5}{2} = \frac{5}{6}$.

$= \frac{5}{6}$

b) $\frac{-4}{9} \cdot \frac{15}{2}$ One negative means the end result is negative. Place a negative sign in front of the first fraction and eliminate the other negative signs.

$= -\frac{4}{9} \cdot \frac{15}{2}$ 9 and 15 simplify by a factor of 3; 4 and 2 simplify by a factor of 2.

$= -\frac{2}{3} \cdot \frac{5}{1}$ Multiply.

$= -\frac{10}{3}$

You Try It 8 Evaluate. Use Example 8 as a guide.

a) $\frac{-2}{3} \cdot \frac{-4}{5}$ b) $-\frac{15}{8} \cdot \frac{4}{3}$ c) $-\frac{10}{7} \cdot \left(-\frac{-14}{5}\right)$

THE RECIPROCAL OF A FRACTION

The *reciprocal* of a fraction, $\frac{a}{b}$, is the inverted fraction, $\frac{b}{a}$. For example, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$. If a fraction is negative, its reciprocal is also negative. For example, the reciprocal of $\frac{-6}{11}$ is $\frac{-11}{6}$.

Whether a fraction is positive or negative, the product of a fraction and its reciprocal is always 1. For example, $\frac{3}{8} \cdot \frac{8}{3} = \frac{24}{24} = 1$, and $\frac{-6}{11} \cdot \frac{-11}{6} = \frac{+66}{66} = 1$.

Example 9: Place the correct number in the blank space to create a product of 1.

a) $\frac{2}{5} \cdot \_\_\_\_ = 1$ b) $\frac{-4}{7} \cdot \_\_\_\_ = 1$ c) $\frac{1}{3} \cdot \_\_\_\_ = 1$ d) $-9 \cdot \_\_\_\_ = 1$

Procedure: In the blank space, place the reciprocal of the product's first number.

Answer: a) $\frac{5}{2}$ b) $\frac{-7}{4}$ c) $\frac{3}{1}$ or 3 d) $\frac{-1}{9}$

You Try It 9 Place the correct number in the blank space to create a product of 1. Use Example 9 as a guide.

a) $\frac{-3}{10} \cdot \_\_\_\_ = 1$ b) $\frac{2}{9} \cdot \_\_\_\_ = 1$ c) $6 \cdot \_\_\_\_ = 1$ d) $\frac{-1}{2} \cdot \_\_\_\_ = 1$