Mathematical Tidbits

➢ **The Distributive Property** ... can only be applied when the outside operation is one rank immediately above the inside operation.

Multiplication over addition

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

Multiplication over subtraction

$$a \cdot (b - c) = a \cdot b - a \cdot c$$

Division under addition

$$(b + c) \div a = b \div a + c \div a$$

$$\frac{b + c}{a} = \frac{b}{a} + \frac{c}{a}$$

Division under subtraction

$$(b - c) \div a = b \div a - c \div a$$

$$\frac{b - c}{a} = \frac{b}{a} - \frac{c}{a}$$

Exponents over Multiplication

$$(a \cdot b)^c = a^c \cdot b^c$$

Exponents over Division

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

Radicals over Multiplication

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

Radicals over Division

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

This law prevents any distribution with $(a + b)^c$: $(a + b)^c \neq a^c + b^c$ Likewise, $\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$

➢ **Division by a fraction:** $\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$ When applying this rule, it can be helpful to create *common denominators* first.

Example: $\frac{5}{8} \div \frac{1}{6}$ LCD = 24:

$$\frac{5}{8} \cdot \frac{3}{3} \div \frac{1}{6} \cdot \frac{4}{4} = \frac{15}{24} \div \frac{4}{24} = \frac{15 \div 4}{24 \div 24} = \frac{15 \div 4}{1} = \frac{15}{4}$$

"Commutativity" in Expressions

Every expression can be preceded by $\underline{0 +}$ or $\underline{1 \times}$.

If the only operations in an expression are of *the same rank,* then the numbers within the expression can be "commuted" as long as the operation in front of each stays with the number.

Example:

$$\begin{aligned} & 7 - 3 \\ =\ & 0 + 7 - 3 \\ =\ & 0 - 3 + 7 \\ =\ & -3 + 7 \\ =\ & 4 \end{aligned}$$

The sign in front of a number belongs to that number.

$$\begin{aligned} & 8 \div 4 \\ =\ & 1 \times 8 \div 4 \\ =\ & 1 \div 4 \times 8 \\ =\ & \frac{1}{4} \times 8 \\ =\ & 2 \end{aligned}$$

The operation in front of a number belongs to that number.

Furthermore expressions with more than one operation, all of the same rank, can be "commuted" within:

$a \div b \div c = a \div c \div b$ and $a \times b \div c = a \div c \times b$

Example:

$$\begin{aligned} & 24 \div 4 \div 2 \\ =\ & 24 \div 2 \div 4 \end{aligned}$$

$$\begin{aligned} & 40 \times 2 \div 4 \\ =\ & 40 \div 4 \times 2 \end{aligned}$$

Why these work:

$$\begin{aligned} & a \div b \div c \\ =\ & a \times \frac{1}{b} \times \frac{1}{c} \\ =\ & a \times \frac{1}{c} \times \frac{1}{b} \\ =\ & a \div c \div b \end{aligned}$$

← convert to fractions →

← commute →

← convert back →

$$\begin{aligned} & a \times b \div c \\ =\ & a \times b \times \frac{1}{c} \\ =\ & a \times \frac{1}{c} \times b \\ =\ & a \div c \times b \end{aligned}$$

Inverse Operations and Distribution

Subtraction is the inverse operation of addition, and division is the inverse operation of multiplication.

With that in mind, consider the following truths:

$$a + (b + c) = a + b + c \qquad a \times (b \times c) = a \times b \times c$$

$$a + (b - c) = a + b - c \qquad a \times (b \div c) = a \times b \div c$$

$$a - (b + c) = a - b - c \qquad a \div (b \times c) = a \div b \div c$$

$$a - (b - c) = a + b + c \qquad a \div (b \div c) = a \div b \times c$$

Notice that division acts a lot like subtraction when it is “distributed.”

Algebra: Solving Equations

We need to be careful with the language we use when speaking to math students. In solving equations, I now regularly use “each side” instead of “both sides” when speaking about isolating the variable.

For example, I have seen students write the following $(x + 1)^2 = 4$

when told to “Take the square root of both sides.” .... $\sqrt{(x+1)^2 = 4}$ *(Wrong!)*

The square root of *each side* is better .... $\sqrt{(x+1)^2} = \sqrt{4}$

.... but we should **NEVER** write .... $\sqrt{(x+1)^2} = \pm\sqrt{4}$ *(W rong!)*

The ± symbol should be reserved for the next step .... $x + 1 = \pm 2$