Quadratic Solution, The Babylonian Method

Taken from:

http://occawlonline.pearsoned.com/bookbind/pubbooks/angel_awl/chapter1/medialib/internet_projects/chapter6/algebra/algebra.html

but it's easier to find it in a Google search: "Chapter 6" "Quadratics in History".

Consider this problem:

A rectangular plot of land is 20 yards longer than it is wide and has an area of 800 square yards. What are the dimensions of the plot?

Here is a modern algebraic solution:

let W = width of rectangle

let L = length of rectangle

$$L = W + 20$$

$$L \cdot W = 800$$

$$(W + 20)\cdot W = 800$$

$$W^2 + 20W - 800 = 0$$

$$(W + 40)(W - 20) = 0$$

$$W = -40 \text{ (not possible) or } W = 20 \text{ (Solution)}$$

So, the width is 20 yards and the length is 40 yards.

Of course, the ancient mathematicians didn't have the luxury of algebraic symbols, so everything was constructed geometrically, under a set of established rules. Here is the Babylonian solution to this problem:

a) find half of the difference between the length and width: $\frac{20}{2} = 10$ yards.

b) Square this value: $10^2 = 100$

c) Add the known area to this square: $100 + 800 = 900$

d) Take the square root of this value: $\sqrt{900} = 30$

e) To this square root, add half of the difference to get L and subtract half the difference to get W:

$L = 30 + 10 = \mathbf{40}$; $W = 30 - 10 = \mathbf{20}$ which is consistent with our modern solution.

Let's see what this looks like algebraically:

$$L = W + d$$
$$L \cdot W = A$$

a) find half of the difference between the length and width: $\frac{d}{2}$

b) Square this value: $\left(\frac{d}{2}\right)^2 = \frac{d^2}{4}$

c) Add the known area to this square: $\frac{d^2}{4} + A$

d) Take the square root of this value: $\sqrt{\frac{d^2}{4} + A}$

e) To this square root, add half of the difference to get L and subtract half the difference to get W:

$$L = \sqrt{\frac{d^2}{4} + A} + \frac{d}{2} \qquad W = \sqrt{\frac{d^2}{4} + A} - \frac{d}{2}$$

$$L = \sqrt{\frac{d^2}{4} + \frac{4A}{4}} + \frac{d}{2} \qquad W = \sqrt{\frac{d^2}{4} + \frac{4A}{4}} - \frac{d}{2}$$

$$L = \sqrt{\frac{d^2 + 4A}{4}} + \frac{d}{2} \qquad W = \sqrt{\frac{d^2 + 4A}{4}} - \frac{d}{2}$$

$$L = \frac{\sqrt{d^2 + 4A}}{2} + \frac{d}{2} \qquad W = \frac{\sqrt{d^2 + 4A}}{2} - \frac{d}{2}$$

$$L = \frac{d + \sqrt{d^2 + 4A}}{2} \qquad W = \frac{- d + \sqrt{d^2 + 4A}}{2}$$

These might be considered "cousins" of the quadratic formula.