Quadratic Equation, A Geometric Solution

Taken from:

http://occawlonline.pearsoned.com/bookbind/pubbooks/angel_awl/chapter1/medialib/internet_projects/chapter6/algebra/algebra.html

but it's easier to find it in a Google search: "Chapter 6" "Quadratics in History".

Consider this problem: A square and 20 roots is 300.

Here, a root is x and the square is x^2. So this says $x^2 + 20x = 300$

On page 2 is a geometric solution, developed by the Middle Eastern mathematician al-Khwarizmi (or Abu Ja'far Muhammad ibn Musa Al-Khwarizmi), believed to have lived from 780 AD to 850 AD. You'll notice that the solution, creative in its design, requires the completion of the square.

You'll also notice that it yields only one solution, $x = 10$, yet our algebraic methods will give us two solutions:

$$x^2 + 20x = 300$$

$$x^2 + 20x - 300 = 0$$

$$(x - 10)(x + 30) = 0$$

$$x = 10, \quad x = -30$$

Since the geometric method considers only positive solutions, it makes sense that $x = -30$ is not a viable solution.

Furthermore, we could get two solutions only if the constant (on the right side of the equation) were negative. It is unlikely, however, that negative constants would be considered at all.

I became curious, though, and asked, "What if the problem read: A square less 20 roots is 300?" How might we approach it then?

Algebraically it would look like this:

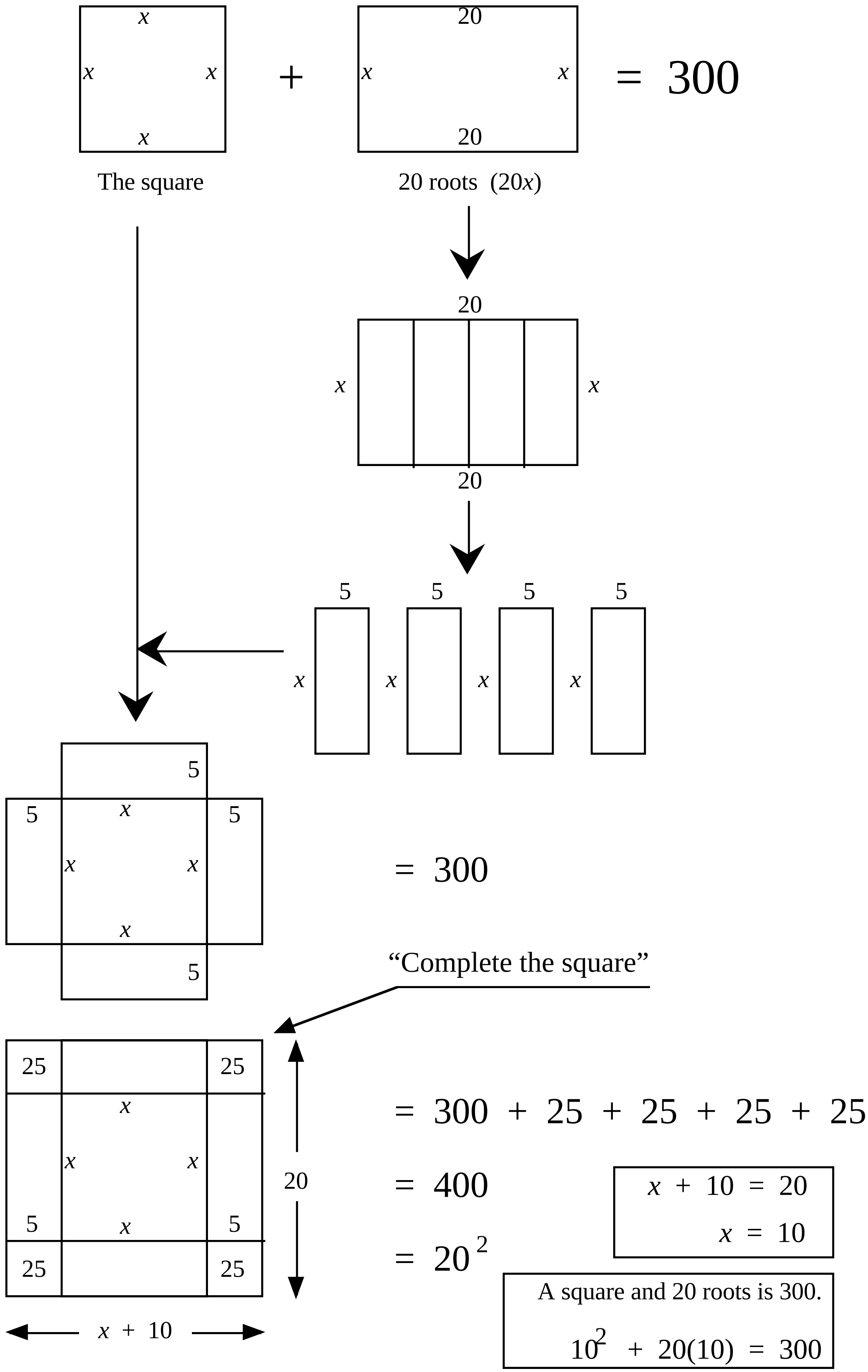
$$x^2 - 20x = 300$$

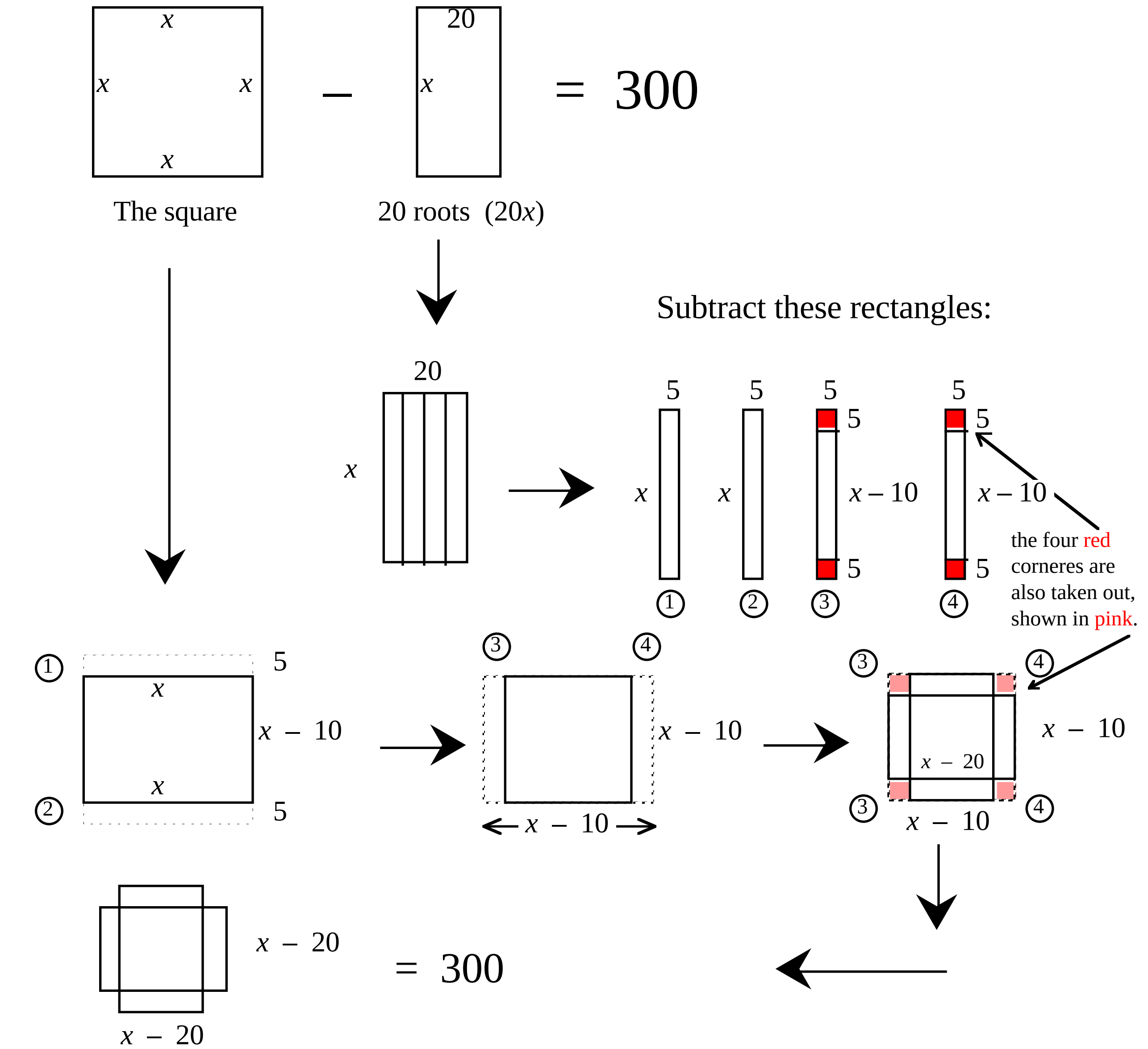
$$x^2 - 20x - 300 = 0$$

$$(x + 10)(x - 30) = 0$$

$$x = -10, \quad x = 30$$

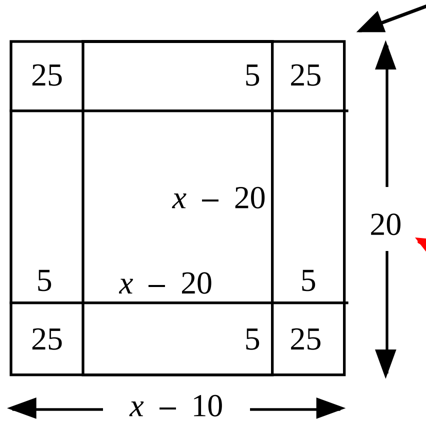
This time, the solution is 30. Geometrically, the solution is shown on page 3.


x
x x
x
The square
+
20
x x
20
20 roots (20x)
= 300
20
x x
20
5 5 5 5
x x x x
5
5 x 5
x x
x
5
= 300
"Complete the square"
25 25
x
x x
5 x 5
25 25
20
x + 10
= 300 + 25 + 25 + 25 + 25
= 400
= 20²
x + 10 = 20
x = 10
A square and 20 roots is 300.
10² + 20(10) = 300



Super-Sized:

"Complete the square"



$$= 300 + 25 + 25 + 25 + 25$$

$$= 400$$

$$= 20^2$$

$$x - 10 = 20$$
$$x = 30$$

A square less 20 roots is 300.

$$30^2 - 20(30) = 300$$