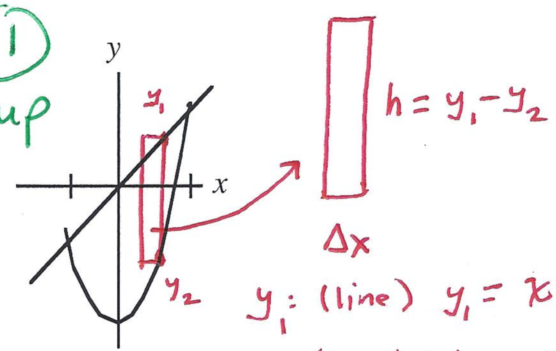
Math 1B **Final Exam Review**

Chapter 6

Find the area of the region bounded by the given curves/lines.

1. $y = x^2 - 6$ and $y = x$

1 the set up



y_1: (line) $y_1 = x$

y_2: (parabola) $y_2 = x^2 - 6$

Points of intersection:

$y_1 = y_2$

$x = x^2 - 6$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3, -2$

2 the integral for Area

$$A = \int_{-2}^{3} [y_1 - y_2]\,dx$$

$$= \int_{-2}^{3} [x - (x^2 - 6)]\,dx$$

$$= \int_{-2}^{3} (x - x^2 + 6)\,dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + 6x \Big|_{-2}^{3}$$

3 the numbers.

$$= \left(\frac{9}{2} - \frac{27}{3} + 18\right) - \left(\frac{4}{2} - \frac{-8}{3} - 12\right)$$

$$= \frac{9}{2} - \frac{4}{2} - \frac{27}{3} - \frac{8}{3} + 18 + 12$$

$$= \frac{5}{2} - \frac{35}{3} + 30$$

$$= 2 + \frac{1}{2} - 11 - \frac{2}{3} + 30$$

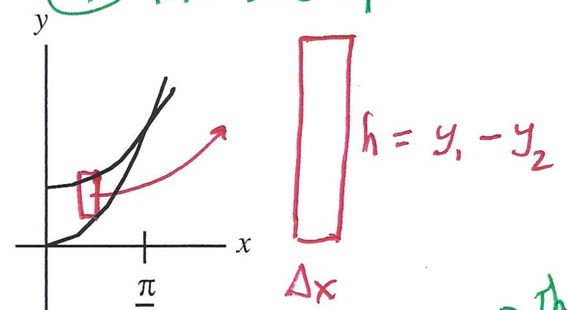
$$= 21 + \frac{3}{6} - \frac{4}{6}$$

$$= 21 - \frac{1}{6} = \boxed{20\frac{5}{6}}$$

there are other ways to evaluate the numbers but they should all lead to the same answer.

2. $y = \sec(x), y = 2\tan(x)$, from $x = 0$ to $x = \frac{\pi}{6}$

① the set up



$y_1 = \sec x$
$y_2 = 2\tan x$
They intersect at $\pi/6$, as shown

$h = \sec x - 2\tan x$

② the integral for area

$$A = \int_0^{\pi/6} (\sec x - 2\tan x)\,dx$$

$$= \ln|\sec x + \tan x| - 2\ln|\sec x| \Big|_0^{\pi/6}$$

(Because sec x and tan x are positive (non-negative) in this region, we don't need absolute value for the rest of our work.)

③ the numbers

$$= \left[\ln\left(\sec\tfrac{\pi}{6} + \tan\tfrac{\pi}{6}\right) - 2\cdot\ln\left(\sec\tfrac{\pi}{6}\right)\right] - \left[\ln(\sec 0 + \tan 0) - 2\ln(\sec 0)\right]$$

$$= \underbrace{\ln\left(\frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\right) - 2\ln\left(\frac{2\sqrt{3}}{3}\right)} - \ln(1+0) + 2\ln(1)$$

$$- \ln(1) \quad + \quad 2\cdot 0$$

$$- 0 \quad + \quad 0$$

From here, there are a variety of ways to simplify. Here is one option:

$$= \ln\left(\frac{3\sqrt{3}}{3}\right) - \ln\left(\frac{2\sqrt{3}}{3}\right)^2$$

$$= \ln(\sqrt{3}) - \ln\left(\frac{4\cdot\cancel{3}^{1}}{\cancel{9}_{3}}\right)$$

$$= \ln\left(\frac{\sqrt{3}}{4/3}\right)$$

$$= \boxed{\ln\left(\frac{3\sqrt{3}}{4}\right)}$$