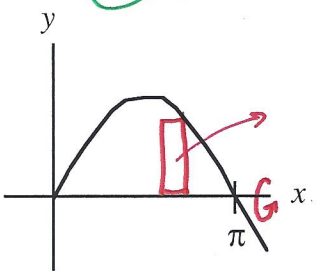
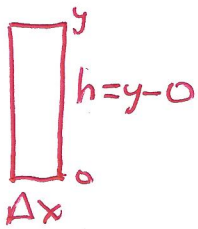
Find the area of the region described. Draw a typical rectangle in the diagram as well as the associated disk, washer or cylindrical shell.

3. $y = \sin x$ and the x-axis, $0 \le x \le \pi$, rotated about the x-axis

(1) the set up





(2) the integral for a disk:

$$V = \pi \int_0^{\pi} r^2 \, dx$$

$$= \pi \int_0^{\pi} \sin^2 x \, dx$$

Power reduction formula for $\sin^2 x$

$$= \pi \int_0^{\pi} \frac{1}{2}\left[1 - \cos(2x)\right] dx$$

$$= \frac{\pi}{2} \int_0^{\pi} \left(1 - \cos(2x)\right) dx$$

$$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin(2x)\right]\Big|_0^{\pi}$$

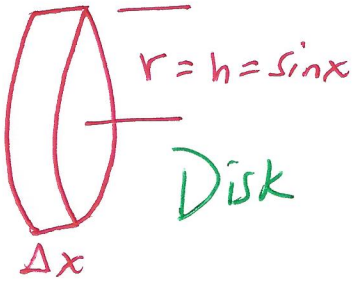
$$= \frac{\pi}{2}\left[\left(\pi - \frac{1}{2}\sin(2\pi)\right) - \left(0 - \frac{1}{2}\sin(0)\right)\right]$$

$$= \frac{\pi}{2}\left[\pi - 0 - 0 - 0\right]$$

$$= \boxed{\frac{\pi^2}{2}}$$

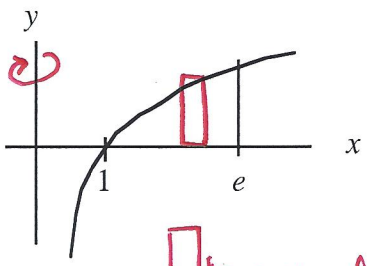
$h = \sin x$

Limits: $[0, \pi]$



4. $y = \ln(x)$, and the x-axis, $1 \le x \le e$, rotated about the y-axis

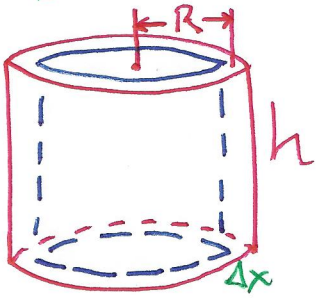
(1) the set up



$h = y = \ln x$

Δx

Shell method (rectangle is parallel to axis of rotation)



$R = x$

$h = \ln x$

(2) the integral for a shell

$$V = 2\pi \int_1^e R\,h\,dx$$

$$= 2\pi \int_1^e x \cdot \ln x\,dx$$

This requires integration by parts:

(Integration by parts: if the integral contains an inverse function, it must be u; everything else is dv.)

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$$= 2\pi \left[\frac{x^2}{2}\ln x \Big|_1^e - \int_1^e \frac{x^2}{2} \cdot \frac{1}{x}dx \right]$$

Separate this into two parts; distribute 2π and solve each separately.

(3) the numbers

$$= \pi x^2 \ln x \Big|_1^e - \pi \int_1^e x\,dx$$

$$= \pi\left[e^2 \ln e - 1^2 \ln 1\right] - \pi \frac{x^2}{2}\Big|_1^e$$

$$= \pi\left[e^2 \cdot 1 - 1 \cdot 0\right] - \pi\left[\frac{e^2}{2} - \frac{1}{2}\right]$$

$$- \frac{\pi e^2}{2} + \frac{\pi}{2}$$

$$= \frac{2\pi e^2}{2}$$

Bring them back together

$$= \boxed{\frac{\pi e^2}{2} + \frac{\pi}{2}} \quad \text{or} \quad \boxed{\frac{\pi}{2}\left[e^2 + 1\right]}$$