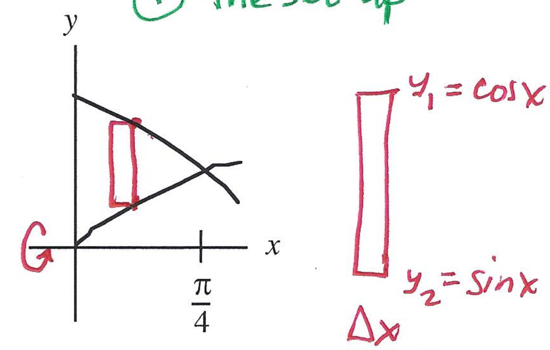
(1) the set up

5. $y = \sin(x)$ and $y = \cos(x)$, $0 \le x \le \frac{\pi}{4}$, rotated about the x-axis



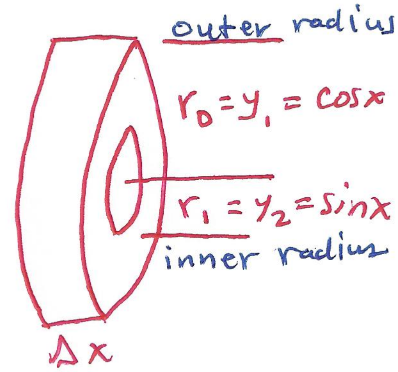
(2) the integral for a washer

$$V = \pi \int_0^{\pi/4} \left[r_o^{\,2} - r_i^{\,2} \right] dx$$

$$= \pi \int_0^{\pi/4} \left[(\cos x)^2 - (\sin x)^2 \right] dx$$

$$= \pi \int_0^{\pi/4} (\cos^2 x - \sin^2 x)\, dx$$

washer method: $\perp$ to axis of rotation but not flush with axis.



(2a) the easy way:

Recognize that $\cos^2 x - \sin^2 x$ is an identity for $\cos(2x)$:

$$= \pi \int_0^{\pi/4} \cos(2x)\, dx$$

$$= \pi \cdot \frac{1}{2} \sin(2x) \Big|_0^{\pi/4}$$

$$= \frac{\pi}{2} \cdot \left[\sin\left(\frac{2\pi}{4}\right) - \sin(0) \right]$$

$$= \frac{\pi}{2} \cdot \sin\left(\frac{\pi}{2}\right) - 0$$

$$= \frac{\pi}{2} \cdot 1 = \boxed{\frac{\pi}{2}}$$

(2b) the longer way: use power reduction formulas on both $\cos^2 x$ and $\sin^2 x$:

$$= \pi \int_0^{\pi/4} \left[\frac{1}{2}(1 + \cos(2x)) \right] - \left[\frac{1}{2}(1 - \cos(2x)) \right] dx$$

Factor out $\frac{1}{2}$

$$= \frac{\pi}{2} \int_0^{\pi/4} \left[1 + \cos(2x) - 1 + \cos(2x) \right] dx$$

$$= \frac{\pi}{2} \int_0^{\pi/4} 2 \cdot \cos(2x)\, dx$$

$$= \pi \int_0^{\pi/4} \cos(2x)\, dx$$

$$= \pi \cdot \frac{1}{2} \sin(2x) \Big|_0^{\pi/4}$$

same

$$= \cdots = \boxed{\frac{\pi}{2}}$$

Chapter 7

Evaluate each using a technique of integration.

6. $\int x^3 e^{x^2}\, dx$

② u-substitution
Let $u = x^2$
$du = 2x\,dx$

$$= \int x^2 \cdot x \cdot e^{x^2} \cdot dx$$

$$= \frac{1}{2}\int x^2 \cdot e^{x^2} \cdot 2x\,dx$$

$$= \frac{1}{2}\int u \cdot e^u\, du$$

Now we use integration by parts:

$w = u$, $dv = e^u du$
$dw = du$ $v = e^u$

$$= \frac{1}{2}\left[ue^u - \int e^u du\right]$$

$$= \frac{1}{2}\left[ue^u - e^u\right]$$

ⓐ we'll put "+ C" at the end.
ⓑ resubstitute for u.

$$= \frac{1}{2}\left[x^2 e^{x^2} - e^{x^2}\right]$$

$$= \boxed{\frac{1}{2} e^{x^2}(x^2 - 1) + C}$$

① Some of these integrals require more than one technique of integration, include u-substitution and integration by parts, as in #6. This means that we must be careful with our choices of substitution variables. For this, and others, I'll use w and v for integration by parts:

$$\int w\,dv = w \cdot v - \int v \cdot dw$$