7. $\int x\tan^{-1}x\,dx$

$$= \underbrace{\frac{1}{2}x^2\tan^{-1}(x)}_{} - \frac{1}{2}\underbrace{\int\frac{x^2}{x^2+1}dx}_{}$$

(a) this is complete

(b) this integral requires more work:

1st, integration by parts:

$u=\tan^{-1}(x) \qquad dv = x\,dx$

$du = \frac{1}{x^2+1}dx \qquad v=\frac{1}{2}x^2$

(some write this denom. as $1+x^2$; either way is fine.)

2nd use long division to simplify the fraction. (we might also be able to use trig-substitution, but that's a bit much right now.)

(b) Consider only $\int\frac{x^2}{x^2+1}dx$

$$= \int\left(1-\frac{1}{x^2+1}\right)dx$$

$$\begin{array}{r} 1 \\ x^2+1\,\overline{)\,x^2+0} \\ -(x^2+1) \\ \hline -1 \end{array}$$

$= 1 - \frac{1}{x^2+1}$

(b) $= x - \tan^{-1}(x)$

putting it all together:

(a) + (b)

$$= \frac{1}{2}x^2\tan^{-1}(x) - \frac{1}{2}\left[x - \tan^{-1}(x)\right]$$

$$= \boxed{\frac{1}{2}x^2\tan^{-1}(x) - \frac{1}{2}x + \frac{1}{2}\tan^{-1}(x)}$$

8. $\int e^{\sqrt{x}}\,dx$ ← this is a little to read because the square root does not print well when it's an exponent: $e^{\sqrt{x}}$

$= \int e^u \cdot 2u\,du$

1st u-substitution:

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$= 2\int e^u \cdot u\,du$

2nd Integration by parts

$w = u \quad dv = e^u du$

$dw = du \quad v = e^u$

$= 2\left[u \cdot e^u - \int e^u du\right]$

$= 2ue^u - 2 \cdot e^u$

3rd resubstitute the value of $u = \sqrt{x}$

$= 2e^u(u-1)$

$= \boxed{2e^{\sqrt{x}}(\sqrt{x}-1) + C}$