Chapter 11 Power Series Practice Test

Answer Key
Name

1. Use the Ratio Test to determine the interval of convergence for x. Be sure to check for endpoint convergence.

a) $S = \sum_{n=1}^{\infty} \frac{(5x)^n}{n^2 4^n}$ ← First write the numerator as $5^n \cdot x^n$ so we can see the separate factors.

$= \sum_{n=1}^{\infty} \frac{5^n \cdot x^n}{n^2 \cdot 4^n}$ Now use the Ratio Test to help identify the interval of convergence.

(1) Ratio Test:

Invert and multiply and simplify the factors:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{5^{n+1}\cdot x^{n+1}}{(n+1)^2\cdot 4^{n+1}}}{\frac{5^n\cdot x^n}{n^2\cdot 4^n}}\right| = \lim_{n\to\infty}\left|\frac{5^n\cdot 5\cdot x^n\cdot x}{(n+1)^2\cdot 4^n\cdot 4}\cdot\frac{n^2\cdot 4^n}{5^n\cdot x^n}\right|$$

$\lim_{n\to\infty}$ and absolute value bars are required throughout.

$$= \lim_{n\to\infty}\left|\frac{5\cdot x}{(n+1)^2\cdot 4}\cdot\frac{n^2}{1}\right| = \lim_{n\to\infty}\left|\frac{5x n^2}{4(n+1)^2}\right| = \left|\frac{5x}{4}\right|\cdot\lim_{n\to\infty}\frac{n^2}{(n+1)^2}$$

$$\lim_{n\to\infty}\frac{n^2}{n^2+\underbrace{2n+1}_{\text{negligible}}} = \lim_{n\to\infty}\frac{n^2}{n^2} = 1$$

$= \left|\frac{5x}{4}\right|\cdot 1 = \left|\frac{5x}{4}\right|$ The ratio test says "Convergent" when less than 1, so .....

$\left|\frac{5x}{4}\right| < 1 \rightarrow |x| < \frac{4}{5}$

(2) initial interval of conv.

(3) Test each endpoint, $x = \frac{4}{5}$, $x = \frac{-4}{5}$:

$x = \frac{4}{5}$, $S = \sum_{n=1}^{\infty}\frac{\left(5\cdot\frac{4}{5}\right)^n}{n^2\cdot 4^n} = \sum_{n=1}^{\infty}\frac{4^n}{n^2\cdot 4^n} = \sum_{n=1}^{\infty}\frac{1}{n^2}$ ← Converges, p-series $p=2$

$x = \frac{-4}{5}$, $S = \sum_{n=1}^{\infty}\frac{\left(5\cdot\frac{-4}{5}\right)^n}{n^2\cdot 4^n} = \sum_{n=1}^{\infty}\frac{(-4)^n}{n^2\cdot 4^n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ ← because this converges as a positive term series, it converges absolutely.

(4) update interval of convergence:

$$\boxed{|x| \le \frac{4}{5}, \text{ or } -\frac{4}{5} \le x < \frac{4}{5}}$$