

4. Find the Taylor Series representation of  $f(x)$ , centered at the given value of  $a$ .

a)  $f(x) = \cos x$ ,  $a = \pi$

$$f(x) = \cos x$$

$$f(\pi) = \cos(\pi) = -1$$

$$f'(x) = -\sin x$$

$$f'(\pi) = -\sin(\pi) = 0$$

$$f''(x) = -\cos x$$

$$f''(\pi) = -\cos(\pi) = +1$$

$$f'''(x) = \sin x$$

$$f'''(\pi) = 0$$

$$f^{(4)}(x) = \cos x$$

$$f^{(4)}(\pi) = -1$$

$$f^{(5)}(x) = -\sin x$$

$$f^{(5)}(\pi) = 0$$

$$f^{(6)}(x) = -\cos x$$

$$f^{(6)}(\pi) = +1$$

So,

$$\begin{aligned} f(x) = \cos x &= -1 + 0 \cdot (x-\pi) + \frac{1 \cdot (x-\pi)^2}{2!} + \frac{0 \cdot (x-\pi)^3}{3!} \\ &+ \frac{-1 \cdot (x-\pi)^4}{4!} + \frac{0 \cdot (x-\pi)^5}{5!} + \frac{1 \cdot (x-\pi)^6}{6!} + \dots \\ &= -1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!} + \dots \end{aligned}$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} \cdot \frac{(x-\pi)^{2n}}{(2n)!}$$