4. Find the Taylor Series representation of $f(x)$, centered at the given value of ~~a~~ c.

b) $f(x) = \frac{1}{x}$, ~~a~~ $c = \frac{1}{2}$

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1x^{-2} = \frac{-1}{x^2} = \frac{-1 \cdot 1!}{x^2}$

$f''(x) = 2x^{-3} = \frac{2}{x^3} = \frac{2!}{x^3}$

$f'''(x) = -6x^{-4} = \frac{-6}{x^4} = \frac{-1 \cdot 3!}{x^4}$

$f^{IV}(x) = 24x^{-5} = \frac{24}{x^5} = \frac{4!}{x^5}$

$f^{(5)}(x) = -120x^{-6} = \frac{-120}{x^6} = \frac{-1 \cdot 5!}{x^6}$

$f(\frac{1}{2}) = \frac{1}{\frac{1}{2}} = 2$

$f'(\frac{1}{2}) = \frac{-1 \cdot 1!}{(\frac{1}{2})^2} = -1 \cdot 2^2$

$f''(\frac{1}{2}) = \frac{2!}{(\frac{1}{2})^3} = 2! \cdot 2^3$

$f'''(\frac{1}{2}) = \frac{-1 \cdot 3!}{(\frac{1}{2})^4} = -1 \cdot 3! \cdot 2^4$

$f^{IV}(\frac{1}{2}) = \frac{+1 \cdot 4!}{(\frac{1}{2})^5} = 1 \cdot 4! \cdot 2^5$

I think I see the pattern

$f^{(n)}(\frac{1}{2}) = (-1)^n \cdot n! \cdot 2^{n+1}$

$$f(x) = \frac{1}{x} = 2 + -1 \cdot 2^2 \cdot (x - \tfrac{1}{2}) + \frac{2! \cdot 2^3 \cdot (x - \frac{1}{2})^2}{2!} + \frac{-1 \cdot 3! \cdot 2^4 (x - \frac{1}{2})^3}{3!}$$

$$= 2^1 - 2^2(x - \tfrac{1}{2})^1 + 2^3(x - \tfrac{1}{2})^2 - 2^4(x - \tfrac{1}{2})^3 + 2^5(x - \tfrac{1}{2})^4 + \dots$$

$$\frac{1}{x} = \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot 2^{n+1} \cdot (x - \tfrac{1}{2})^n}$$

If desired, we can simplify a little further by recognizing

(i) $2 \cdot (x - \frac{1}{2}) = (2x - 1)$

(ii) $2^n (x - \frac{1}{2})^n = [2(x - \frac{1}{2})]^n = (2x-1)^n$

So $2^{n+1}(x - \frac{1}{2})^n = 2 \cdot 2^n (x - \frac{1}{2})^n = 2 \cdot (2x-1)^n$

So, $\frac{1}{x} = \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot 2 \cdot (2x-1)^n}$ Alternative form