1. Use the Ratio Test to determine the interval of convergence for x. Be sure to check for endpoint convergence.

b) $S = \sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n}$

① Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x+1)^{n+1}}{3^{n+1}}}{\frac{(x+1)^n}{3^n}}\right| = \lim_{n\to\infty}\left|\frac{(x+1)^n\cdot(x+1)}{3^n\cdot 3}\cdot\frac{3^n}{(x+1)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x+1}{3}\cdot\frac{1}{1}\right| = \lim_{n\to\infty}\left|\frac{x+1}{3}\right|$$

converges when less than 1, so...

② initial interval of convergence:

$$\left|\frac{x+1}{3}\right| < 1$$

$$-1 < \frac{x+1}{3} < 1$$

$$-3 < x+1 < 3$$

$$-4 < x < 2$$

③ Test endpoints, $x=-4$ and $x=2$:

$x=2$: $S = \sum_{n=0}^{\infty}\frac{(2+1)^n}{3^n} = \sum_{n=0}\frac{3^n}{3^n} = \sum_{n=0} 1 \to \infty$ diverges

$x=-4$: $S = \sum_{n=0}^{\infty}\frac{(-4+1)^n}{3^n} = \sum_{n=0}^{\infty}\frac{(-3)^n}{3^n} = \sum_{n=0}^{\infty}(-1)^n = 1-1+1-1+1\ldots$

diverges b/c it doesn't approach any single number.

④ No change in itial interval of convergence: $\boxed{-4 < x < 2}$ or $\boxed{(-4,2)}$