1. Use the Ratio Test to determine the interval of convergence for x. Be sure to check for endpoint convergence.

absolute value eliminates $(-1)^n$ and $(-1)^{n+1}$, but it is still need for x.

c) $S = \sum_{n=0}^{\infty} (-1)^n \frac{(x-2)^n}{n+1}$

① Ratio Test: $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \cdot \frac{(x-2)^{n+1}}{(n+1)+1}}{(-1)^n \cdot \frac{(x-2)^n}{n+1}} \right| = \lim_{n\to\infty} \left| \frac{(x-2)^n \cdot (x-2)}{n+2} \cdot \frac{n+1}{(x-2)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{(x-2)}{n+2} \cdot \frac{n+1}{1} \right| = \lim_{n\to\infty} \left| \frac{(x-2)(n+1)}{(n+2)} \right|$

$(x-2)$ is unaffected by the limit and can be factored out.

$= |x-2| \cdot \lim_{n\to\infty} \frac{n+1}{n+2} = |x-2| \cdot \lim_{n\to\infty} \frac{n}{n} = |x-2| \cdot 1$

the +1 and +2 are negligible as $n \to \infty$

$= |x-2|$ ② S converges when this is less than 1

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$ initial interval of convergence

③ Test endpoints: $x=1$, $x=3$

$x=1$: $S = \sum_{n=0}^{\infty} (-1)^n \frac{(1-2)^n}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (-1)^n}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{n+1}$

$= \sum_{n=0}^{\infty} \frac{[(-1)^2]^n}{n+1} = \sum_{n=0}^{\infty} \frac{1^n}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges, it is compared to harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$

$x=3$: $S = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (3-2)^n}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 1^n}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

this is compared to "alternating harmonic" series which converges.

④ Updated Interval of Convergence:

$\boxed{1 < x \le 3}$ or $\boxed{(1,3]}$