2. For a Power Series that represents a function, the Foundation Series is

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots + x^n + \ldots$$

with an interval of convergence of $|x| < 1$ or (-1, 1).

Using this Foundation power Series, find each of the following.

(i) Write the answer in both summation (sigma) notation and
(ii) expanded form (at least four non-zero terms) and
~~(iii) identify the interval of convergence.~~ Note: I'm eliminating this from #2.

a) $f(x) = \dfrac{x}{1+x^2}$

(1) First find power series for $g(x) = \dfrac{1}{1+x^2} = \dfrac{1}{1-(-x^2)} =$ put $(-x^2)$ into the Foundation series $= \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n \cdot (x^2)^n$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n}$$

(2) now multiply by x to get $f(x) = \dfrac{x}{1+x^2} = x \cdot \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n} = \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n} \cdot x$

(i) $= \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot x^{2n+1}}$

(ii) $= \boxed{x - x^3 + x^5 - x^7 + \cdots}$