2c) Based on your answer to part **(a)**, find the power series representation for

$f(x) = \ln(1 + x^2)$

$$\frac{x}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n \cdot x^{2n+1}$$

First recognize:

$$f'(x) = \frac{1}{1+x^2} \cdot 2x$$

$$f'(x) = \frac{2x}{1+x^2}$$

To match this up with $f'(x)$ at left, multiply each side by 2:

$$\frac{2 \cdot x}{1+x^2} = 2 \cdot \sum_{n=0}^{\infty}(-1)^n \cdot x^{2n+1}$$

Now take the integral of each side to go from $f'(x)$ to $f(x)$:

$$\int \frac{2x}{1+x^2}\,dx = \int 2 \cdot \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n+1}\,dx$$

On the rightside, only x^{2n+1} is affected by the integral, so we can write it as:

$$= 2 \cdot \sum_{n=0}^{\infty}(-1)^n \cdot \int x^{2n+1}\,dx$$

power rule

$$= 2 \sum_{n=0}^{\infty}(-1)^n \cdot \frac{x^{(2n+1)+1}}{(2n+1)+1}$$

bring the 2 inside and simplify the fraction:

$$= \sum_{n=0}^{\infty}(-1)^n \cdot 2 \cdot \frac{x^{2n+2}}{2n+2}$$

factor out 2 and divide out with the other 2.

$$= \sum_{n=0}^{\infty}(-1)^n \cdot 2 \cdot \frac{x^{2n+2}}{2(n+1)}$$

$$= \boxed{\sum_{n=0}^{\infty}(-1)^n \cdot \frac{x^{2n+2}}{n+1}}$$

Now show it in expanded form:

n=0: n=1: n=2: n=3:

$$+\frac{x^2}{1} + (-1)\frac{x^4}{2} + +\frac{x^6}{3} + (-1)\frac{x^8}{4}$$

$$\boxed{= x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots}$$