2d) Based on the answer to part 2b), find the Power Series representation for

$$f(x) = \frac{1}{(x+2)^2}$$

First 2b): $y = \frac{4}{x+2} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^n}{2^{n-1}}$

the connection between these two functions is the derivative:

$y' = \frac{-4}{(x+2)^2}$, which is almost $f(x) = \frac{1}{(x+2)^2}$. Let's work with this y' and adjust the answer at the end so it looks like $f(x)$.

Start again:

$$y = \frac{4}{x+2} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^n}{2^{n-1}}$$

take the derivative

$$y' = \frac{-4}{(x+2)^2} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{2^{n-1}} \cdot \frac{d(x^n)}{dx}$$

Notice that the $\sum$ and $\frac{1}{2^{n-1}}$ are unaffected by the derivative.

$$\frac{-4}{(x+2)^2} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{2^{n-1}} \cdot n \cdot x^{n-1}$$

At this point, to find $f(x)$, we must multiply each side by $\frac{-1}{4}$:

$$\frac{-1}{4} \cdot \frac{-4}{(x+2)^2} = \frac{-1}{4} \cdot \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{2^{n-1}} \cdot n \cdot x^{n-1}$$

Let's bring the $\frac{-1}{4}$ into the $\sum$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{(-1)^1}{2^2} \cdot \frac{1}{2^{n-1}} \cdot n \cdot x^{n-1}$$

this is $\frac{-1}{4}$; Combine with other factors.

$$= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n x^{n-1}}{2^{n+1}}}$$

notice that n now starts at 1 instead of 0. This is because when n=0, the term is 0.

(ii) Expanded form:

$$= +\frac{1 \cdot x^0}{2^2} + \frac{-1 \cdot 2 \cdot x^1}{2^3} + \frac{+1 \cdot 3 \cdot x^2}{2^4} + \frac{-1 \cdot 4 \cdot x^3}{2^5}$$

$$= \boxed{\frac{1}{4} - \frac{2x}{8} + \frac{3x^2}{16} - \frac{4x^3}{32} + \cdots}$$

notice that I didn't simplify the 2nd and 4th terms so we can see the pattern.