Taylor Series:

If $f(x)$ can be represented by a power series $\sum_{n=0}^{\infty} a_n \frac{(x-c)^n}{n!}$ with Radius of Convergence, R, then

$$f(x) = f(c) + f'(c)(x-c) + f''(c)\frac{(x-c)^2}{2!} + f'''(c)\frac{(x-c)^3}{3!} + \dots + f^{(n)}(c)\frac{(x-c)^n}{n!} + \dots$$

3. Find the Taylor Series representation of each function, centered at $a = 0$. OOPS! this is supposed to be $\underline{c = 0}$

a) $f(x) = \cos(2x)$

$f(x) = \cos(2x)$ $\quad f(0) = \cos(0) = 1$

$f'(x) = -2\sin(2x)$ $\quad f'(0) = -2\sin(0) = -2\cdot 0 = 0$

$f''(x) = -4\cos(2x)$ $\quad f''(0) = -4\cos(0) = -4\cdot 1 = -4$

$f'''(x) = 8\sin(2x)$ $\quad f'''(0) = 8\sin(0) = 0$

$f^{iv}(x) = 16\cos(2x)$ $\quad f^{iv}(0) = 16\cdot\cos(0) = 16$

$f^{(5)}(x) = -32\sin(2x)$ $\quad f^{(5)}(0) = -32\sin(0) = 0$

$f^{(6)}(x) = -64\cos(2x)$ $\quad f^{(6)}(x) = -64\cos(0) = -64$

So, $f(x) = \cos(2x) = 1 + 0\cdot x + (-4)\cdot\frac{x^2}{2!} + 0\cdot\frac{x^3}{3!} + 16\cdot\frac{x^4}{4!} + 0\cdot\frac{x^5}{5!} + -64\cdot\frac{x^6}{6!} + \dots$

$$= 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \frac{64x^6}{6!} + \dots + (-1)^n\cdot\frac{2^{2n}\cdot x^{2n}}{(2n)!} + \dots$$

$$= \boxed{\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^{2n}\cdot x^{2n}}{(2n)!}} \text{ or } \boxed{\sum_{n=0}^{\infty}(-1)^n\frac{(2x)^{2n}}{(2n)!}}$$