3. Find the Taylor Series representation of each function, centered at $a = 0$.

this should be $c=0$.

We must use the Product Rule for the derivatives.

b) $f(x) = x \sin x$

$$f'(x) = x \cdot \cos x + \sin x \cdot 1 = x\cos x + \sin x$$

$$f''(x) = x \cdot (-\sin x) + \cos x \cdot 1 + \cos x$$
$$= -x\sin x + 2\cos x$$

$$f'''(x) = -(x\cos x + \sin x) - 2\sin x$$
$$= -x\cos x - 3\sin x$$

$$f^{iv}(x) = -(-x\sin x + \cos x) - 3\cos x$$
$$= x\sin x - 4\cos x$$

$$f^{(5)}(x) = (x\cos x + \sin x) + 4\sin x$$
$$= x\cos x + 5\sin x$$

$$f^{(6)}(x) = (-x\sin x + \cos x) + 5\cos x$$
$$= -x\sin x + 6\cos x$$

Derivative observation

① $\dfrac{d(x \cdot \sin x)}{dx} = x\cos x + \sin x$

② $\dfrac{d(x \cos x)}{dx} = -x \cdot \sin x + \cos x$

we'll use these multiple times throughout this process.

Put $x=0$ into each:

$f(0) = 0 \cdot 0 = 0$

$f'(0) = 0 \cdot 1 + 0 = 0$

$f''(0) = 0 \cdot 0 + 2 \cdot 1 = 2$

$f'''(0) = 0 \cdot 1 - 3 \cdot 0 = 0$

$f^{iv}(0) = 0 \cdot 0 - 4 \cdot 1 = -4$

$f^{(5)}(0) = 0 \cdot 1 + 5 \cdot 0 = 0$

$f^{(6)}(0) = -0 \cdot 0 + 6 \cdot 1 = 6$

I think I see the pattern *

* So,

$$f(x) = x \cdot \sin x = 0 + 0 \cdot x + 2 \cdot \frac{x^2}{2!} + 0 \cdot \frac{x^3}{3!} + -4 \cdot \frac{x^4}{4!} + 0\frac{x^5}{5!} + \frac{6x^6}{6!} + 0\frac{x^7}{7!} + \frac{-8x^8}{8!} + \dots$$

$$= \frac{2x^2}{2!} - \frac{4x^4}{4!} + \frac{6x^6}{6!} - \frac{8x^8}{8!} + \dots$$

Note: $\dfrac{8}{8!} = \dfrac{8}{8 \cdot 7!} = \dfrac{1}{7!}$ we can simplify each fraction

$$= \frac{x^2}{1!} - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \dots$$

$$= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{x^{2n}}{2n-1}}$$