Math 1B

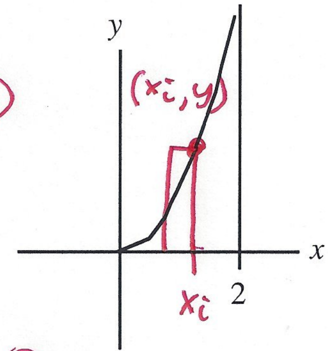
Chapter 6 Pretest

6.1 Area under a curve

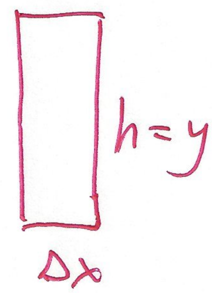
1. Consider the region bounded by the functions $y = x^3$, $x = 2$, and $y = 0$ (as shown). Find the area of this region by doing each of the following:

a) draw a typical rectangle, both inside of and outside of the region;

a



a



b) develop a formula for A_i;

$$A_i = y \cdot \Delta x$$
$$= x_i^3 \cdot \Delta x$$

c) write A as the limit of a sum of areas and simplify;

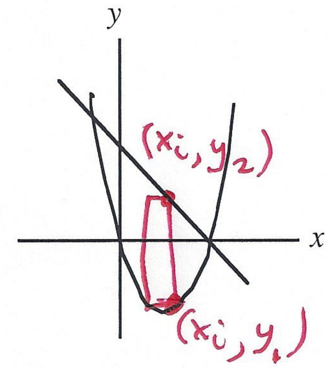
$$A = \lim_{n \to \infty} \sum_{i=1}^{n} x_i^3 \Delta x$$

d) and e) write the related integral and solve it.

$$A = \int_0^2 x^3 \, dx$$
$$= \left. \frac{x^4}{4} \right|_0^2$$
$$= \frac{2^4}{4} - \frac{0}{4}$$
$$= \frac{16}{4}$$
$$= \boxed{4 \text{ units}^2}$$

Find the area of the region bounded by the given curves/lines.

2. $y_1 = x^2 - 4x$ and $y_2 = 4 - x$



One thing we must do at some point is find the limits of integration. Let's do that first:

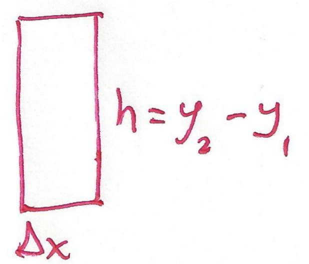
$$y_1 = y_2$$

$$x^2 - 4x = 4 - x$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4, -1$$



$$A = \int_{-1}^{4} (-x^2 + 3x + 4)\,dx$$

$$= -\frac{x^3}{3} + \frac{3x^2}{2} + 4x \Big|_{-1}^{4}$$

$$= \left[-\frac{(4)^3}{3} + \frac{3(4)^2}{2} + 4(4)\right] - \left[-\frac{(-1)^3}{3} + \frac{3(-1)^2}{2} + 4(-1)\right]$$

$$= -\frac{64}{3} + \frac{48}{2} + 16 - \left(\frac{1}{3} + \frac{3}{2} - 4\right)$$

$$= -\frac{65}{3} + \frac{45}{2} + 20$$

$$= \frac{-130 + 135 + 120}{6}$$

$$= \frac{125}{6} \longrightarrow \boxed{A = \frac{125}{6} \text{ units}^2}$$

$$A_i = (y_2 - y_1)\Delta x$$

$$= [(4 - x) - (x^2 - 4x)]\Delta x$$

$$= [4 - x - x^2 + 4x)\Delta x$$

$$= (-x^2 + 3x + 4)\Delta x$$