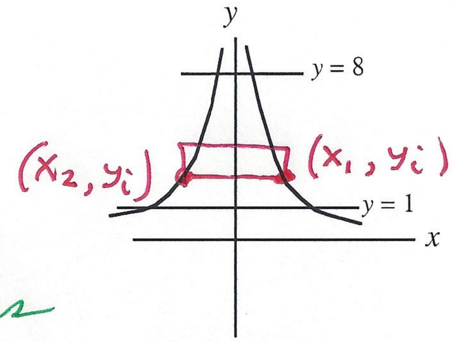
3. $y = \frac{1}{x^3}$, $y = \frac{-1}{x}$, $y = 1$, and $y = 8$

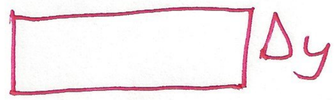
x_1 $\quad$ x_2

Because we have Δy, we must write each equation in terms of y. Solve for each x:



$$y = \frac{1}{x^3}$$
$$y \cdot x^3 = 1$$
$$x^3 = \frac{1}{y}$$
$$x_1 = \sqrt[3]{\frac{1}{y}} = y^{-1/3}$$

$$y = \frac{-1}{x}$$
$$xy = -1$$
$$x_2 = \frac{-1}{y}$$
$$x_2 = -\frac{1}{y}$$



$h = x_1 - x_2$

$$A_i = (x_1 - x_2)\Delta y$$
$$= \left[y^{-1/3} - \left(-\frac{1}{y}\right)\right]\Delta y$$
$$= y^{-1/3} + \frac{1}{y}$$

$$A = \int_1^8 \left(y^{-1/3} + \frac{1}{y}\right) dy$$

$$= \frac{3}{2} \cdot y^{2/3} + \ln|y| \Big|_1^8$$

$$= \left[\frac{3}{2}(8)^{2/3} + \ln(8)\right] - \left[\frac{3}{2}(1)^{2/3} + \ln(1)\right]$$

$$= \frac{3}{2} \cdot 4 + \ln 8 - \frac{3}{2} - 0$$

Notes:
$\ln(1) = 0$
$8^{2/3} = \sqrt[3]{8}^2 = 2^2 = 4$

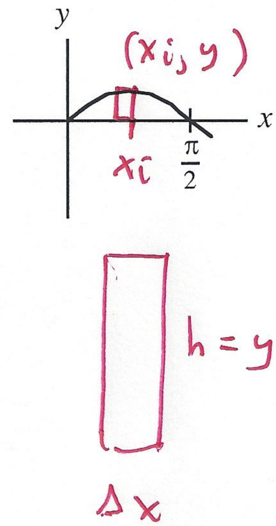
$$= \frac{12}{2} + \ln 8 - \frac{3}{2}$$

$$= \boxed{\frac{9}{2} + \ln 8 \text{ units}^2}$$

This answer could also be written as (one example)

$= 4.5 + 3\ln 2$ units^2

4. $y = \sin x \cos x$, $y = 0$, from $x = 0$ to $x = \frac{\pi}{2}$



$$A_i = y \cdot \Delta x$$

$$= \sin x \cdot \cos x \, \Delta x$$

$$A = \int_0^{\pi/2} \sin x \cdot \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

x	u
$\pi/2$	1
0	0

$$= \int_0^1 u \, du$$

$$= \frac{u^2}{2} \Big|_0^1 = \frac{1}{2} - \frac{0}{2}$$

$$= \boxed{\frac{1}{2} \text{ units}^2}$$