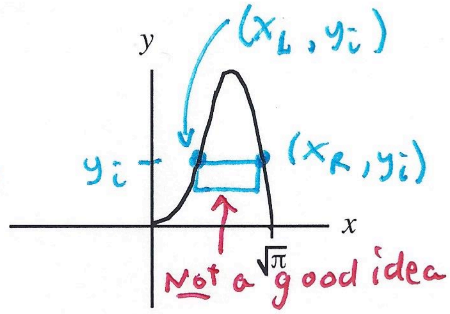
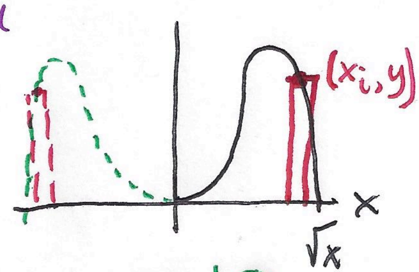
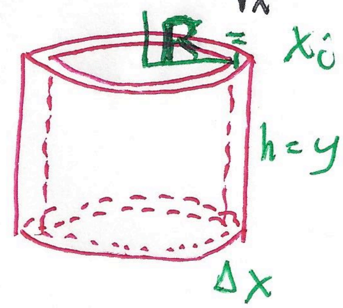
7. $y = \sin(x^2)$, $y = 0$, from $x = 0$ to $x = \sqrt{\pi}$, rotated about the y-axis.



First, with the graph at right, we should be able to see not to use horizontal rectangles because it will be the graph subtracted from itself (right − left).

Instead, we'll use vertical rectangles and rotate them about the y-axis, creating a cylindrical shell, as shown below it. (The reason we get a cylindrical shell is, the vertical rectangles are parallel to the axis of rotation.)





$$V_i = 2\pi R \cdot h \cdot \Delta x = 2\pi \cdot x_i \cdot y \cdot \Delta x$$

$$V = 2\pi \int_0^{\sqrt{\pi}} x \cdot y \cdot dx = 2\pi \int_0^{\sqrt{\pi}} x \cdot \sin(x^2)\, dx$$

← let $u = x^2$

$du = 2x\,dx$

x	u
$\sqrt{\pi}$	π
0	0

$$= \frac{2\pi}{2} \int_0^{\sqrt{\pi}} 2x \cdot \sin(x^2)\, dx$$

$$= \pi \int_0^{\pi} \sin(u)\, du$$

$$= \pi \cdot (-\cos u) \Big|_0^{\pi}$$

$$= -\pi \cdot \cos u \Big|_0^{\pi}$$

$$= -\pi \left[\cos \pi - \cos 0\right]$$

$$= -\pi \left[-1 - 1\right]$$

$$= -\pi(-2)$$

$$= 2\pi$$

Volume:

$$\boxed{V = 2\pi \text{ units}^3}$$

8. Consider the region bounded by the functions $y = \dfrac{4}{x}$ and $y = 5 - x$ (as shown). Use both the washer and cylindrical shell methods to find the volume of the region rotated about the y-axis.

Washer Method

x_1:

$y = \dfrac{4}{x}$

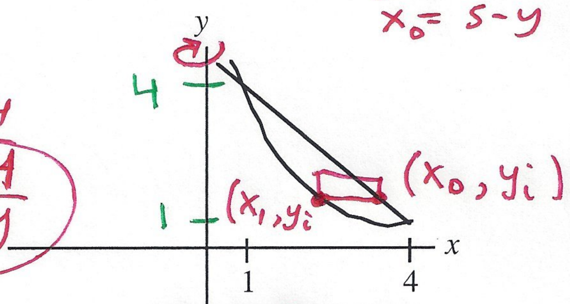
$y \cdot x = 4$

$x_1 = \dfrac{4}{y}$

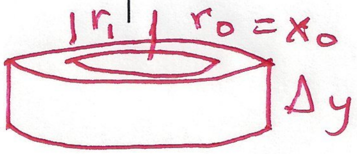
x_0:

$y = 5 - x$

$x_0 = 5 - y$



WASHER:



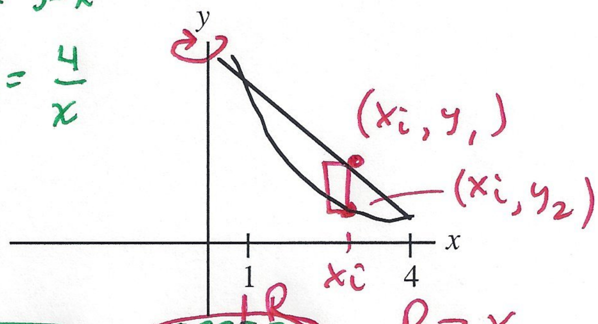
$$V_i = \pi[r_0^2 - r_1^2]\Delta y$$
$$= \pi[x_0^2 - x_1^2]\Delta y$$
$$= \pi\left[(5-y)^2 - \left(\frac{4}{y}\right)^2\right]\Delta y$$
$$= \pi\left[25 - 10y + y^2 - \frac{16}{y^2}\right]\Delta y$$

$$V = \pi\int_1^4 (25 - 10y + y^2 - 16y^{-2})\,dy$$
$$= \pi\left[25y - 5y^2 + \frac{y^3}{3} - \frac{16y^{-1}}{-1}\right]\Bigg|_1^4$$
$$= \pi\left[25y - 5y^2 + \frac{y^3}{3} + \frac{16}{y}\right]\Bigg|_1^4$$
$$= \pi\left[\left(100 - 80 + \frac{64}{3} + 4\right) - \left(25 - 5 + \frac{1}{3} + 16\right)\right]$$
$$= \pi\left[24 + \frac{64}{3} - 25 + 5 - \frac{1}{3} - 16\right]$$
$$= \pi\left[-12 + \frac{63}{3}\right]$$
$$= \pi[-12 + 21] = \boxed{9\pi \text{ units}^3}$$

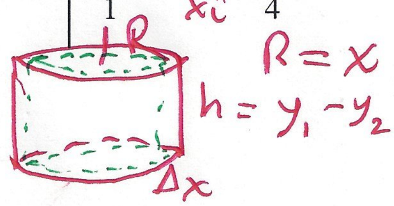
Cylindrical Shell Method

$y_1 = 5 - x$

$y_2 = \dfrac{4}{x}$



SHELL:



$$V_i = 2\pi R h \Delta x$$
$$= 2\pi x \cdot (y_1 - y_2)\Delta x$$
$$= 2\pi x\left(5 - x - \frac{4}{x}\right)\Delta x$$
$$= 2\pi(5x - x^2 - 4)\Delta x$$

$$V = 2\pi\int_1^4 (5x - x^2 - 4)\,dx$$
$$= 2\pi\left[\frac{5x^2}{2} - \frac{x^3}{3} - 4x\right]\Bigg|_1^4$$
$$= 2\pi\left[\left(\frac{80}{2} - \frac{64}{3} - 16\right) - \left(\frac{5}{2} - \frac{1}{3} - 4\right)\right]$$
$$= 2\pi\left[40 - \frac{64}{3} - 16 - \frac{5}{2} + \frac{1}{3} + 4\right]$$
$$= 2\pi\left[24 - \frac{63}{3} - \frac{5}{2} + 4\right]$$
$$= 2\pi\left[28 - 21 - \frac{5}{2}\right]$$
$$= 2\pi\left[7 - \frac{5}{2}\right]$$
$$= 2\pi\left[\frac{14-5}{2}\right]$$
$$= 2\pi \cdot \frac{9}{2} = \boxed{9\pi \text{ units}^3}$$