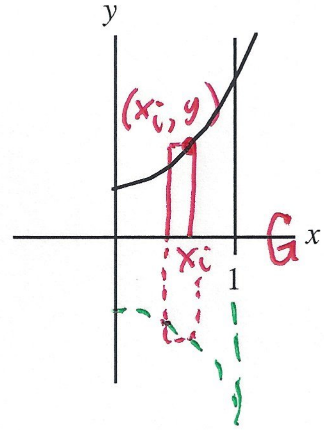
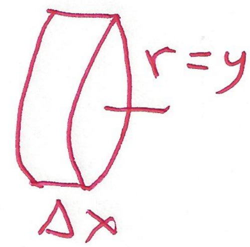
## 6.2 & 6.3 Volumes of Solids of Revolution

**5.** $y = e^x$, $y = 0$, from $x = 0$ to $x = 1$, rotated about the $x$-axis.




$$V_i = \pi r^2 \Delta x$$
$$= \pi (y)^2 \Delta x$$
$$= \pi (e^{2x}) \Delta x$$

Disk




$$V = \pi \int_0^1 e^{2x}\, dx$$

$$= \pi \cdot \frac{1}{2} e^{2x} \Big|_0^1$$

$$= \frac{\pi}{2} e^2 - \frac{\pi}{2} e^0$$

$$= \boxed{\frac{\pi}{2}\left(e^2 - 1\right) \text{ units}^3}$$

Because $2x$ is "linear" we can get the antiderivative without having to use substitution:

$$\int e^{2x}\, dx = \frac{1}{2} e^{2x}$$

However, using substitution is okay, too.

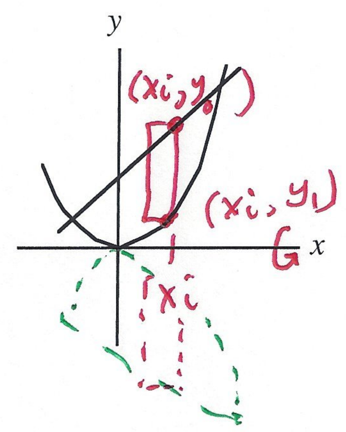
$u = 2x$

$du = 2\, dx$

$\vdots$

and so on.

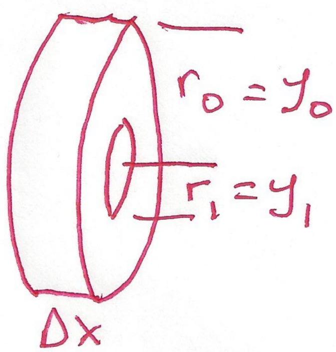
**6.** $\overbrace{y = x^2}^{y_1}$, $\overbrace{y = x + 2}^{y_o}$, rotated about the $x$-axis.




Limits of integration:

$$y_1 = y_o$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2, -1$$

$$V_i = \pi\left(r_o^2 - r_1^2\right)\Delta x$$
$$= \pi\left[y_o^2 - y_1^2\right]\Delta x$$
$$= \pi\left[(x+2)^2 - (x^2)^2\right]\Delta x$$
$$= \pi\left[x^2 + 4x + 4 - x^4\right]\Delta x$$

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$$V = \pi\int_{-1}^{2}\left(x^2 + 4x + 4 - x^4\right)dx$$
$$= \pi\left[\frac{x^3}{3} + 2x^2 + 4x - \frac{x^5}{5}\right]\Bigg|_{-1}^{2}$$
$$= \pi\left[\left(\frac{8}{3} + 8 + 8 - \frac{32}{5}\right) - \left(\frac{-1}{3} + 2 - 4 + \frac{1}{5}\right)\right]$$
$$= \pi\left[\frac{9}{3} + 16 - \frac{33}{5} - 2 + 4\right]$$
$$= \pi\left[3 + 16 - 6 - \frac{3}{5} + 2\right]$$
$$= \pi\left[15 - \frac{3}{5}\right]$$
$$= \pi\left[\frac{75 - 3}{5}\right] = \boxed{\frac{72}{5}\pi \text{ units}^3}$$