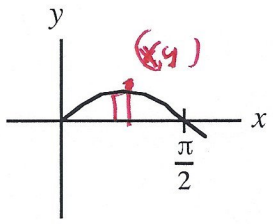
3. $y = \sin(x)\cos(x)$, $y = 0$, from $x = 0$ to $x = \frac{\pi}{2}$



$$A = \int_0^{\pi/2} \sin x \cdot \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

x	u
$\pi/2$	$\sin\frac{\pi}{2} = 1$
0	$\sin 0 = 0$

$$= \int_0^1 u \, du$$

$$= \frac{u^2}{2}\Big|_0^1 = \frac{1}{2} - 0$$

$$= \boxed{\frac{1}{2}}$$

h = y

Δx

6.4 Work stretching a spring

For the work of stretching a spring, write all answers in foot-pounds (ft-lb).

4. A spring has a natural length of 12 inches. A force of 150 lb is required to stretch the spring to 21 inches (9 inches beyond its natural length).

a) What is the spring constant, k?

$f(9 \text{ in.}) = f(\frac{9}{12} \text{ ft})$

Hooke's Law: $f(x) = kx$

x = distance stretched beyond natural length

$= f(\frac{3}{4}) = k \cdot \frac{3}{4} = 150$

$\frac{4}{3} \cdot \frac{3}{4} k = 150 \cdot \frac{4}{3}$

$k = 200$

b) How much work is done in stretching the spring from

i) 12 inches to 18 inches? (x = 0 in. to x = 6 in.)

$$W = \int_0^{1/2} 200x \, dx$$

$$= 100x^2\Big|_0^{1/2}$$

$$= 100\left(\frac{1}{2}\right)^2 - 0$$

$$= 100 \cdot \frac{1}{4} = \boxed{25 \text{ ft-lb}}$$

ii) 15 inches to 21 inches? (x = 3 in. to x = 9 in.)

$$W = \int_{1/4}^{3/4} 200x \, dx$$

$$= 100x^2\Big|_{1/4}^{3/4}$$

$$= 100\left[\left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right]$$

$$= 100\left(\frac{9}{16} - \frac{1}{16}\right) = 100\left(\frac{8}{16}\right)$$

$$= \boxed{50 \text{ ft-lb}}$$