5. The work required to stretch a spring 6 inches beyond its natural length is 90 ft-lb.

a) What is the spring constant, k?

$$W = \int_0^{1/2} kx\,dx$$

$$= \frac{kx^2}{2}\Big|_0^{1/2}$$

$$= \frac{k\left(\frac{1}{2}\right)^2}{2} - 0$$

$$= \frac{1}{2}k \cdot \frac{1}{4}$$

$$= \frac{1}{8}k = 90$$

$$k = 720$$

b) How much work is done in stretching the spring from 6 inches to 10 inches beyond its natural length?

$6 \text{ in} = \frac{6}{12} = \frac{1}{2}$ ft

$10 \text{ in} = \frac{10}{12} = \frac{5}{6}$ ft

$$W = \int_{1/2}^{5/6} 720x\,dx$$

$$= \frac{720x^2}{2}\Big|_{1/2}^{5/6}$$

$$= 360\left[\left(\frac{5}{6}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

$$= 360\left[\frac{25}{36} - \frac{1}{4}\cdot\frac{9}{9}\right]$$

$$= 360\left[\frac{25-9}{36}\right]$$

$$= \frac{360}{1}\cdot\frac{16}{36}$$

$$= 10 \cdot 16$$

$$= 160 \text{ ft-lb}$$

6. A spring with constant $k = 120$, requires 15 ft-lb of work to stretch the spring from 7 inches to 8 inches. What is the spring's natural length? *(Two hints: (i) Let n = natural length of the spring; and (ii) … hmmm, what's the other hint?)*

Note: This type of problem will not be on the test.