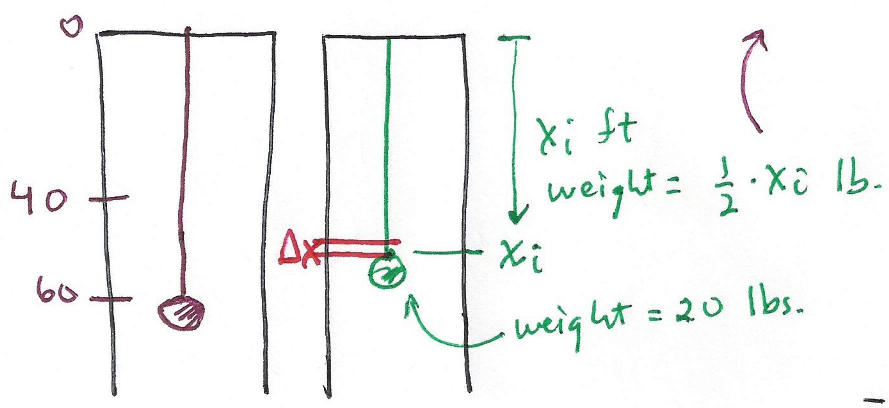
6.4 Work lifting a weight

7. A 20-lb weight is being raised by a 60-foot rope weighing $\frac{1}{2}$ lb/ft. Determine the work needed to lift the weight 20 feet.



$$\text{Force}_i = \frac{1}{2}x_i + 20$$

$$W_i = F_i \cdot \Delta x = \left(\frac{1}{2}x_i + 20\right)\Delta x$$

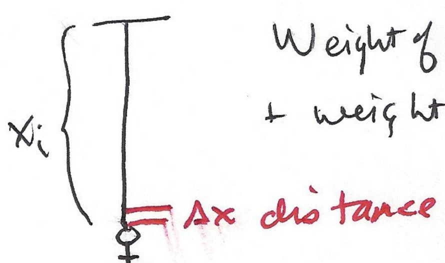
$$W = \int_{40}^{60} \left(\frac{1}{2}x + 20\right) dx$$

$$= \frac{1}{2}\frac{x^2}{2} + 20x \Big|_{40}^{60}$$

$$= \left[\frac{1}{4}(60)^2 + 20(60)\right] - \left[\frac{1}{4}(40)^2 + 20(40)\right]$$

$$= \frac{3600}{4} + 1200 - \frac{1600}{4} - 800 = \cdots = \boxed{900 \text{ ft-lb}}$$

8. A ships anchor weighs 1,000 pounds and the anchor chain weighs 50 lb/ft. What is the work done in pulling up the anchor if 100 feet of chain are out?



Weight of chain $= 50x_i$
+ weight of anchor $= 1{,}000$ } Force$_i$

Force$_i$ × distance

$$W_i = (50x_i + 1000) \times \Delta x$$

$$W = \int_0^{100} (50x + 1000)\, dx$$

$$= 25x^2 + 1000x \Big|_0^{100}$$

$$= 25(100)^2 + 1000(100)$$

$$= 250{,}000 + 100{,}000$$

$$= \boxed{350{,}000 \text{ ft-lb}}$$