Math 1B **Chapter 7 Pre-Test (Sec. 7.1 - 7.5)** Answers

Name

Evaluate each using integration by parts.

1. $\int x^2 e^x \, dx$ ← (1)

$u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$= x^2 e^x - 2\int x e^x dx$ ← (2)

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$

$= x^2 e^x - 2\left[x e^x - e^x\right] + C$

$= \boxed{x^2 e^x - 2x e^x + 2e^x + C} = \boxed{e^x\left(x^2 - 2x + 2\right) + C}$

2. $\int x^{-3} \ln x \, dx$

$u = \ln x \quad dv = x^{-3} dx$

$du = \frac{1}{x} dx \quad v = \frac{x^{-2}}{-2}$

$= \ln x\left(-\frac{1}{2} x^{-2}\right) - \int -\frac{1}{2} x^{-2} \cdot \frac{1}{x} dx$

$= \ln x\left(-\frac{1}{2} x^{-2}\right) + \frac{1}{2}\int x^{-3} dx$

$= \ln x\left(-\frac{1}{2} x^{-2}\right) + \frac{1}{2} \cdot \frac{x^{-2}}{-2} + C$

$= \boxed{-\frac{1}{2} x^{-2}\left[\ln x + \frac{1}{2}\right] + C}$ or $\boxed{\frac{-1}{4x^2}\left[2\ln x + 1\right] + C}$

3. $\int e^{-x}\sin x\,dx$

① $u = e^{-x}$ $dv = \sin x\,dx$

$du = -e^{-x}dx$ $v = -\cos x$

$= -e^{-x}\cos x + \int e^{-x}\cos x\,dx$

② $u = e^{-x}$ $dv = \cos x\,dx$

$du = -e^{-x}dx$ $v = \sin x$

$= -e^{-x}\cos x - \left[e^{-x}\sin x - \int e^{-x}\sin x\,dx\right]$

$\int e^{-x}\sin x\,dx = -e^{-x}\cos x - e^{-x}\sin x + \int e^{-x}\sin x\,dx$

(add this term to each side)

$2\int e^{-x}\sin x\,dx = -e^{-x}(\cos x + \sin x)$

$$\int e^{-x}\sin x\,dx = -\frac{1}{2}e^{-x}(\cos x + \sin x) + C$$

Note: we could also choose $u = \sin x$, $dv = e^{-x}dx$

Evaluate each trigonometric integral.

Let's first simplify the integrand:

$(\sec x + \tan x)^2$

$= \sec^2 x + 2\sec x\tan x + \tan^2 x$

$= \sec^2 x + 2\sec x\tan x + \sec^2 x - 1$

$= 2\sec^2 x + 2\sec x\tan x - 1$

4. $\int (\sec x + \tan x)^2\,dx$

$= \int (2\sec^2 x + 2\sec x\tan x - 1)\,dx$

$= 2\int \sec^2 x\,dx + 2\int \sec x\tan x\,dx - \int dx$

$= 2\tan x + 2\sec x - x + C$

5. $\int \cos^3 x \, dx$

$= \int \cos^2 x \cdot \cos x \, dx$

$= \int (1 - \sin^2 x) \cdot \cos x \, dx$ $\quad u = \sin x$, $du = \cos x \, dx$

$= \int (1 - u^2) \, du$

$= u - \frac{1}{3} u^3 + C$

$= \boxed{\sin x - \frac{1}{3} \sin^3 x + C}$

6. $\int \sec^4 x \, dx$

$= \int \sec^2 x \cdot \sec^2 x \, dx$

$= \int (\tan^2 x + 1) \sec^2 x \, dx$ $\quad u = \tan x$, $du = \sec^2 x \, dx$

$= \int (u^2 + 1) \, du$

$= \frac{1}{3} u^3 + u + C$

$= \boxed{\frac{1}{3} \tan^3 x + \tan x + C}$