Here is #49 (altered slightly) from the Table of Integrals. Use a technique of integration to show that this is true.

12. $\displaystyle\int \frac{dx}{x(nx+k)} = \frac{1}{k}\ln\left|\frac{x}{nx+k}\right| + C$

$$\int \frac{1}{x\cdot(nx+k)}\,dx$$

use partial fractions to rewrite the function

$$\frac{1}{x(nx+k)} = \frac{A}{x} + \frac{B}{nx+k}$$

$$0x + 1 = A(nx+k) + Bx$$

$$0x + 1 = Anx + Ak + Bx$$

(1) x-terms: $0 = An + B$

(2) constants: $1 = A\cdot k \implies A = \frac{1}{k}$

(1): $0 = \frac{1}{k}\cdot n + B$

$-\frac{n}{k} = B$

So the integral becomes

$$\int\left[\frac{\frac{1}{k}}{x} - \frac{\frac{n}{k}}{nx+k}\right]dx = \frac{1}{k}\int\frac{1}{x}\,dx - \frac{n}{k}\int\frac{1}{nx+k}\,dx$$

$$= \frac{1}{k}\ln|x| - \frac{n}{k}\cdot\frac{1}{n}\ln|nx+k| + C$$

Factor out $\frac{1}{k}$ and combine the logs:

$$= \frac{1}{k}\ln|x| - \frac{1}{k}\ln|nx+k| + C$$

$$= \boxed{\frac{1}{k}\ln\left|\frac{x}{nx+k}\right| + C}$$

Evaluate each using a familiar but unusual technique of integration.

13. $\int \frac{1}{2-\sqrt{x}}\,dx$

the familiar technique is u-substitution; what is unusual about it is what we do with the value of u.

Let u equal the square root

$u = \sqrt{x}$ square each side.

$u^2 = x$ now find the derivative to get dx

$2u\,du = dx$

Let's put the pieces of the integral in terms of u.

$$\int \frac{1}{2-\sqrt{x}}\,dx = \int \frac{1}{2-u}\cdot 2u\,du = \int \frac{2u}{2-u}\,du$$

At this point, I believe it is better to have the denominator in descending order and with a positive lead term.

$$\int \frac{2u}{2-u}\,du = \int \frac{-1}{-1}\cdot\frac{2u}{(-u+2)}\,du = -2\int \frac{u}{u-2}\,du$$

long division:

$$\begin{array}{r} 1 + \frac{2}{u-2} \\ u-2 \overline{)\, u + 0} \\ -(u-2) \\ \hline 2 \end{array}$$

$$= -2\int\left[1 + \frac{2}{u-2}\right]du = -2\int 1\,du - 4\int \frac{1}{u-2}\,du$$

$$= -2u - 4\ln|u-2| + C = \boxed{-2\sqrt{x} - 4\ln|\sqrt{x}-2| + C}$$