Evaluate each using trigonometric substitution.

**7.** $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$

$$= \int \frac{1}{x^2} \cdot \frac{1}{\sqrt{4-x^2}} \cdot dx$$

$$= \int \left(\frac{1}{x}\right)^2 \cdot \frac{1}{\sqrt{4-x^2}} \cdot dx$$

$$= \int \left(\frac{1}{2}\sec\theta\right)^2 \cdot \frac{1}{2}\csc\theta \cdot (-2)\sin\theta\, d\theta$$

$$= \int \frac{1}{4}\sec^2\theta \cdot \frac{1}{2} \cdot \csc\theta \cdot (-2)\sin\theta\, d\theta$$

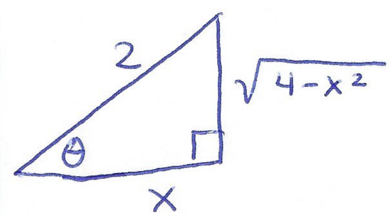
$$= -\frac{1}{4}\int \sec^2\theta \cdot \frac{1}{\sin\theta} \cdot \sin\theta\, d\theta$$

$$= -\frac{1}{4}\int \sec^2\theta\, d\theta$$

$$= -\frac{1}{4}\tan\theta + C$$

$$= -\frac{1}{4} \cdot \frac{\sqrt{4-x^2}}{x} + C$$

$$= \boxed{\frac{-\sqrt{4-x^2}}{4x} + C}$$




$\sec\theta = \frac{2}{x}$

$\frac{1}{2}\sec\theta = \frac{1}{x}$

$\csc\theta = \frac{2}{\sqrt{4-x^2}}$

$\frac{1}{2}\csc\theta = \frac{1}{\sqrt{4-x^2}}$

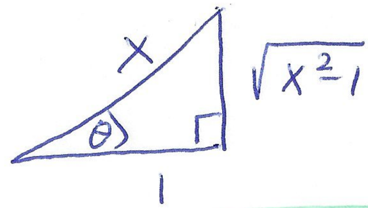
$\cos\theta = \frac{x}{2}$

$2\cos\theta = x$

$-2\sin\theta\, d\theta = dx$

$\tan\theta = \frac{\sqrt{4-x^2}}{x}$

**8.** $\displaystyle\int \frac{\sqrt{x^2-1}}{x}\,dx$




$$\sin\theta = \frac{\sqrt{x^2-1}}{x}$$

$$\sec\theta = \frac{x}{1}$$

$$\sec\theta = x$$

$$\sec\theta\tan\theta\,d\theta = dx$$

$$= \int \sin\theta \cdot \sec\theta \cdot \tan\theta\,d\theta$$

$$= \int \sin\theta \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}\,d\theta$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$

$$= \int \tan^2\theta\,d\theta$$

$$= \int (\sec^2\theta - 1)\,d\theta$$

$$= \tan\theta - \theta + C$$

$$\tan\theta = \sqrt{x^2-1}$$

$$\sec\theta = x$$

so, $\theta = \sec^{-1}(x)$

$$= \boxed{\sqrt{x^2-1} - \sec^{-1}(x) + C}$$