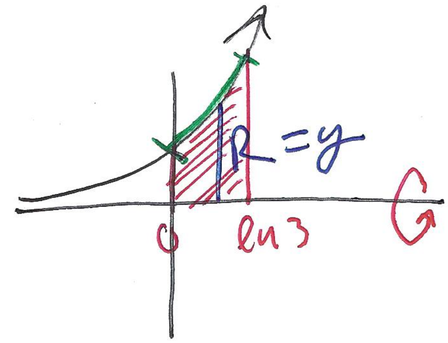
Find the area of the surface of revolution.

10. $y = \sqrt{e^x + 1}$, for $0 \le x \le \ln 3$, rotated about the x-axis



$$y' = \frac{1}{2}(e^x+1)^{-1/2} \cdot e^x$$

$$= \frac{e^x}{2\sqrt{e^x+1}}$$

$$A = 2\pi \int_0^{\ln 3} y \cdot \sqrt{1 + \left[\frac{e^x}{2\sqrt{e^x+1}}\right]^2}\, dx$$

$$= 2\pi \int_0^{\ln 3} \sqrt{e^x+1} \cdot \sqrt{1 + \frac{e^{2x}}{4(e^x+1)}}\, dx$$

$$= 2\pi \int_0^{\ln 3} \sqrt{e^x+1} \sqrt{\frac{4(e^x+1)+e^{2x}}{4(e^x+1)}}\, dx$$

$$= 2\pi \int_0^{\ln 3} \cancel{\sqrt{e^x+1}} \cdot \frac{\sqrt{4e^x+4+e^{2x}}}{2\cancel{\sqrt{e^x+1}}}\, dx$$

Inside, the radicand is "Quadratic in form": if we think of $w = e^x$, the radicand is

$$w^2 + 4w + 4 = (w+2)^2 = (e^x+2)^2$$

$$= \frac{2\pi}{2} \int_0^{\ln 3} \sqrt{e^{2x}+4e^x+4}\, dx$$

$$= \pi \int_0^{\ln 3} \sqrt{(e^x+2)^2}\, dx$$

$$= \pi \int_0^{\ln 3} (e^x+2)\, dx = \pi\left(e^x + 2x\right)\Big|_0^{\ln 3}$$

$$= \pi\left[\left(e^{\ln 3} + 2\ln 3\right) - \left(e^0 + 2\cdot 0\right)\right]$$

$$= \pi\left[3 + 2\ln 3 - 1 - 0\right]$$

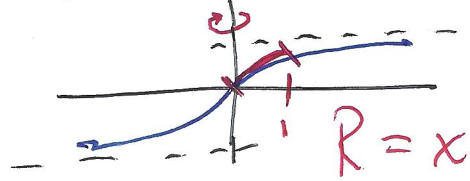
$$= \pi\left[2 + 2\ln 3\right]$$

$$= \boxed{2\pi(1+\ln 3)}$$

11. $y = \tan^{-1}x$, for $0 \le x \le 1$, rotated about the y-axis

Note the limits of x.

graph of $f(x) = \tan^{-1}(x)$



$$y' = \frac{1}{x^2+1}$$

$$A = 2\pi \int_0^1 x \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2}\, dx$$

$$= 2\pi \int_0^1 x \sqrt{1 + \frac{1}{(x^2+1)^2}}\, dx$$

$$= \frac{2\pi}{2} \int_0^1 2x \sqrt{\frac{(x^2+1)^2+1}{(x^2+1)^2}}\, dx$$

Let $u = x^2+1$
$du = 2x\,dx$

x	y
1	2
0	1

$$= \pi \int_1^2 \frac{\sqrt{u^2+1}}{\sqrt{u^2}}\, du$$

$$= \pi \int_1^2 \frac{\sqrt{u^2+1}}{u}\, du$$

This fits an integral pattern on page 1 of this Review:

$$\int \frac{\sqrt{x^2+a^2}}{x}\, dx = \dots$$

In this case x is u and a is 1

$$= \pi \left[\sqrt{u^2+1} - 1 \cdot \ln\left| \frac{1+\sqrt{x^2+1}}{x} \right| \right]_1^2$$

$$= \pi \cdot \left[\left(\sqrt{4+1} - \ln\left| \frac{1+\sqrt{4+1}}{2} \right| \right) - \left(\sqrt{1+1} - \ln\left| \frac{1+\sqrt{1+1}}{1} \right| \right) \right]$$

$$= \pi \cdot \left[\sqrt{5} - \ln\frac{1+\sqrt{5}}{2} - \sqrt{2} + \ln\frac{1+\sqrt{2}}{1} \right]$$

This doesn't simplify very well.

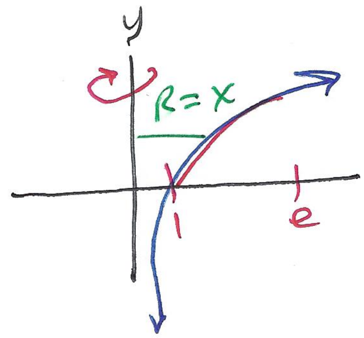
$$= \boxed{\pi \left(\sqrt{5} - \sqrt{2} + \ln(1+\sqrt{2}) - \ln\left(\frac{1+\sqrt{5}}{2}\right) \right)}$$

$$= \boxed{\pi \left(\sqrt{5} - \sqrt{2} + \ln\frac{(1+\sqrt{2})\cdot 2}{1+\sqrt{5}} \right)}$$

Other forms may be acceptable.

Set up and simplify—but do not solve—the integral that represents the area of the surface of revolution of the function on the given interval.

12. $y = \ln x$, for $1 \le x \le e$, rotated about the y-axis



$$y' = \frac{1}{x}$$

$$A = 2\pi \int_1^e x\sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx$$

$$= 2\pi \int_1^e x\sqrt{1 + \frac{1}{x^2}}\, dx$$

$$= 2\pi \int_1^e x\sqrt{\frac{x^2+1}{x^2}}\, dx$$

$$= 2\pi \int_1^e x \cdot \frac{\sqrt{x^2+1}}{x}\, dx$$

$$= \boxed{2\pi \int_1^e \sqrt{x^2+1}\, dx}$$

This integral function can be evaluated using the Table of integrals. It is initially developed ~~from~~ using trig-substitution.