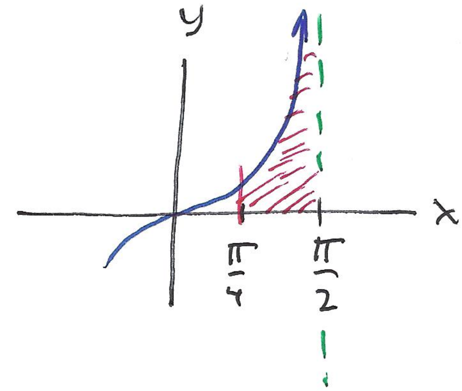
5. $\displaystyle\int_{\pi/4}^{\pi/2} \tan x \, dx$



$$= \lim_{t \to \frac{\pi}{2}^{+}} \int_{\pi/4}^{t} \tan x \, dx$$

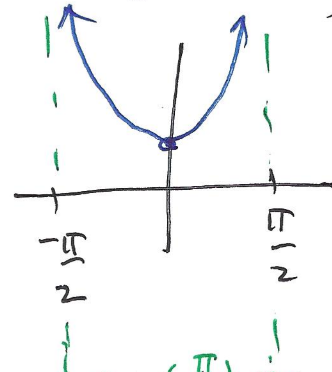
$$= \lim_{t \to \frac{\pi}{2}^{-}} \ln|\sec x| \Big|_{\pi/4}^{t}$$

$$= \lim_{t \to \frac{\pi}{2}^{-}} \left(\ln|\sec t| - \ln\left|\sec \tfrac{\pi}{4}\right| \right)$$

$$= \infty - \sqrt{2}$$

$$= \infty \qquad \boxed{\text{diverges}}$$

graph of $\sec x = y$



$\sec(\frac{\pi}{2})$ ~~is~~ undefined.

6. $\displaystyle\int_3^4 \frac{1}{x\sqrt{4-x}}\,dx$

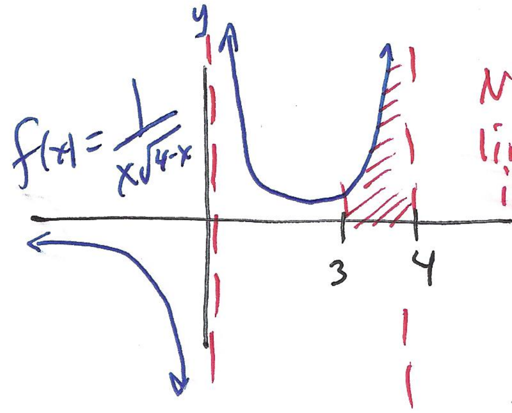
Let $u = \sqrt{4-x}$

$u^2 = 4 - x$

$x = 4 - u^2$

$dx = -2u\,du$

x	u
4	0
3	1



Note the limits of integration.

$$= \int_1^0 \frac{1}{(4-u^2)\cdot \cancel{u}} \cdot (-2)\cancel{u}\,du$$

$$= -2\int_1^0 \frac{1}{4-u^2}\,du$$

$$= +2\int_0^1 \frac{1}{4-u^2}\,du$$

$$= 2\int_0^1 \frac{1}{-(u^2-4)}\,du$$

$$= -2\int_0^1 \frac{1}{u^2-4}\,du$$

use partial fractions here:

$$\frac{1}{u^2-4} = \frac{1}{(u-2)(u+2)} = \frac{A}{u-2} + \frac{B}{u+2}$$

$$1 = A(u+2) + B(u-2)$$

$u=-2$: $1 = B(-4)$, $B = -\frac{1}{4}$

$u=2$: $1 = 4A$, $A = \frac{1}{4}$

$$= -2\int_0^1 \left(\frac{\frac{1}{4}}{u-2} + \frac{-\frac{1}{4}}{u+2}\right)du$$

$$= -2\cdot\frac{1}{4}\int_0^1 \left(\frac{1}{u-2} + \frac{-1}{u+2}\right)du$$

$$= -\frac{1}{2}\Big[\ln|u-2| - \ln|u+2|\Big]\Big|_0^1$$

$$= -\frac{1}{2}\ln\left|\frac{u-2}{u+2}\right|\Big|_0^1$$

$$= -\frac{1}{2}\left(\ln\left|\frac{1-2}{1+2}\right| - \ln\left|\frac{0-2}{0+2}\right|\right)$$

$$= -\frac{1}{2}\left[\ln\left|\frac{1}{3}\right| - \ln\left|\frac{-2}{2}\right|\right]$$

$$= -\frac{1}{2}\left(\ln\frac{1}{3} - \ln 1\right)$$

$$= -\frac{1}{2}\ln\frac{1}{3} - 0$$

$$= \frac{1}{2}\ln\left(\frac{1}{3}\right)^{-1}$$

$$= \boxed{\frac{1}{2}\ln 3} \text{ or } \boxed{\ln\sqrt{3}}$$

CONVERGES