Find the arc length of the function on the given interval.

7. $y = \frac{2}{3}(x-1)^{3/2} + 4$, for $2 \le x \le 5$

$y' = \frac{2}{3} \cdot \frac{3}{2}(x-1)^{1/2} \cdot 1$

$y' = \sqrt{x-1}$

$L = \int_2^5 \sqrt{1 + (\sqrt{x-1})^2}\, dx$

$= \int_2^5 \sqrt{1 + x - 1}\, dx$

$= \int_2^5 \sqrt{x}\, dx$

$= \frac{2}{3} \cdot x^{3/2} \Big|_2^5$

$= \frac{2}{3}\left[5^{3/2} - 2^{3/2}\right]$

$= \frac{2}{3}\left[5\sqrt{5} - 2\sqrt{2}\right]$

Note: $a^{3/2} = a^{2/2} \cdot a^{1/2}$

$= a \cdot a^{1/2}$

$= a\sqrt{a}$

Set up and simplify—but do not solve—the integral that represents the arc length of the function on the given interval.

8. $y = (x-1)^{1/2}$, for $5 \le x \le 10$

$$y' = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}}$$

$$L = \int_5^{10} \sqrt{1 + \left(\frac{1}{2\sqrt{x-1}}\right)^2}\,dx = \int_5^{10} \sqrt{1 + \frac{1}{4(x-1)}}\,dx$$

$$= \int_5^{10} \sqrt{\frac{4(x-1)+1}{4(x-1)}}\,dx = \int_5^{10} \sqrt{\frac{4x-3}{4(x-1)}}\,dx$$

factor out $\sqrt{4}$ in the denom.

$$= \boxed{\frac{1}{2}\int_5^{10} \sqrt{\frac{4x-3}{x-1}}\,dx}$$

9. $y = \ln(x-1)^2$, for $2 \le x \le 3$

first, the exponent of 2 can be placed in front of the log: $\boxed{y = 2 \cdot \ln(x-1)}$ now find y'.

$$y' = 2 \cdot \frac{1}{x-1} = \frac{2}{x-1}$$

$$L = \int_2^3 \sqrt{1 + \left(\frac{2}{x-1}\right)^2}\,dx$$

$$= \int_2^3 \sqrt{1 + \frac{4}{(x-1)^2}}\,dx$$

$$= \int_2^3 \sqrt{\frac{(x-1)^2 + 4}{(x-1)^2}}\,dx$$

$$= \int_2^3 \sqrt{\frac{x^2-2x+1+4}{(x-1)^2}}\,dx$$

$$= \int_2^3 \sqrt{\frac{x^2-2x+5}{(x-1)^2}}\,dx$$

$$= \boxed{\int_2^3 \frac{\sqrt{x^2-2x+5}}{x-1}\,dx}$$

other forms may be acceptable.