#7 Continued: $x = \frac{3}{t}$ and $y = t^2 - 6t$, for $t \geq 1$.

d) Find the second derivative, $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}}$$

$$= \frac{-2t^2 + 4t}{\frac{-3}{t^2}}$$

$$= \frac{-t^2}{3}(-2t^2 + 4t)$$

$$= \boxed{\frac{2}{3}t^4 - \frac{4}{3}t^3}$$

e) Determine the intervals of concavity for the curve.

$$\frac{2}{3}t^4 - \frac{4}{3}t^3$$

$$= \frac{2}{3}t^3(t-2)$$

because $t \geq 1$ $\frac{2}{3}t^3$ is always positive. Concavity, then, depends only on the value of $t-2$.

Concave down:

$t - 2 < 0$

$t < 2$

$[1, 2)$

Concave up:

$t - 2 > 0$

$t > 2$

$(2, \infty)$

f) Use the information above and a table of values for *t*, *x*, and *y*, and sketch the curve in the *x*-*y*-plane.
(Hint: for *t* use integers) $t \geq 1$

t	$x = \frac{3}{t}$	$y = t^2 - 6t$	
1	3	-5	
2	3/2	-8	point of inflection
3	1	-9	horizontal tangent line
4	3/4	-8	
5	3/5	-5	
6	1/2	0	
7	3/7	7