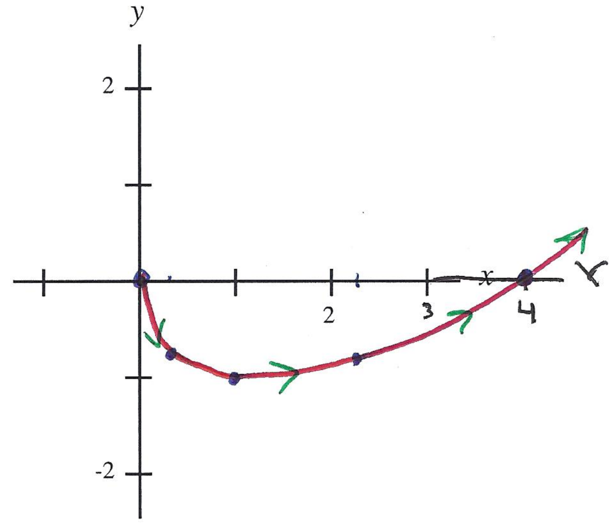
**8.** Consider the curve given by $x = t^2$ and $y = t^2 - 2t$, $t \geq 0$.

a) Set up a table of values for $t$, $x$, and $y$, and sketch the curve in the $x$-$y$-plane, especially where it creates a pocket with the $x$-axis.




| $t$ | $x$ | $t^2 - 2t$ |
|---|---|---|
| 0 | 0 | 0 |
| $1/2$ | $1/4$ | $\frac{1}{4} - 1 = -3/4$ |
| 1 | 1 | $1 - 2 = -1$ |
| $3/2$ | $9/4$ | $\frac{9}{4} - 3 = -3/4$ |
| 2 | 4 | $4 - 4 = 0$ |
| $5/2$ | $25/4$ | $\frac{25}{4} - 5 = +5/4$ |

b) Find the area of the region enclosed between the $x$-axis and the curve.


$$A = \lim_{n \to \infty} \sum_{i=1}^{n} y_i \Delta x$$

not required

this is supposed to be $(0 - y_i) = -y_i$

$$A = -\int_0^2 y \frac{dx}{dt}\, dt$$

$\frac{dx}{dt} = 2t$

$$= -\int_0^2 (t^2 - 2t) 2t\, dt$$

$$= -\int_0^2 (2t^3 - 4t^2)\, dt$$

$$= -\left(\frac{2t^4}{4} - \frac{4t^3}{3}\right)\Bigg|_0^2$$

$$= -\left[\frac{32}{4} - \frac{32}{3}\right] - -(0 - 0)$$

$$= -32\left[\frac{1}{4} - \frac{1}{3}\right]$$

$$= -32\left(-\frac{1}{12}\right) = \frac{32}{12} = \boxed{\frac{8}{3}}$$

c) Set up the integral, and simplify the integrand (but do not solve) for the arc length of the curve from $0 \leq t \leq 2$.

$$L = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^2 \sqrt{(2t)^2 + (2t - 2)^2}\, dt$$

$$= \int_0^2 \sqrt{4t^2 + 4t^2 - 8t + 4}\, dt$$

$$= \int_0^2 \sqrt{8t^2 - 8t + 4}\, dt$$

$$= \boxed{\int_0^2 2\sqrt{2t^2 - 2t + 1}\, dt}$$