Section 10.3

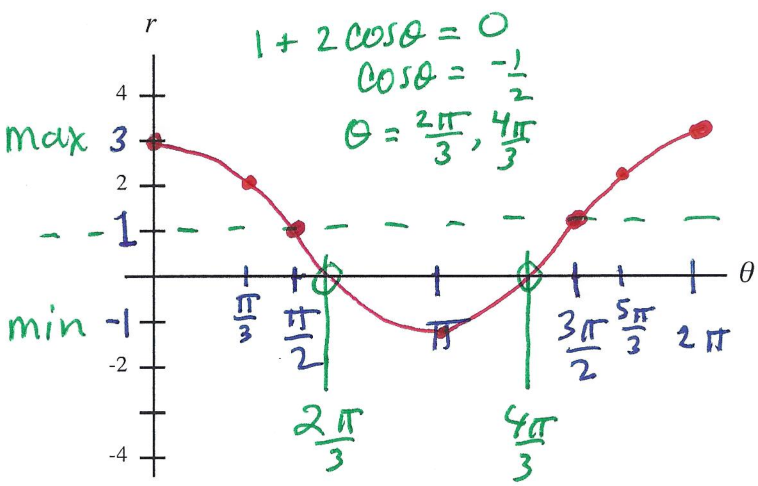
Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in the Cartesian coordinate system.

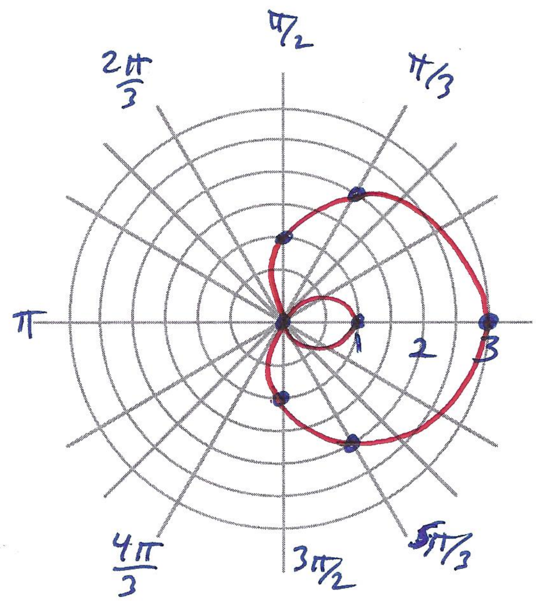
9. $r = 1 + 2\cos\theta$ $\left\{\begin{array}{l} \text{ampl.} = 2 \\ \text{per} = 2\pi \\ \text{shift up } 1 \end{array}\right.$

$1 + 2\cos\theta = 0$

$\cos\theta = \frac{-1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$





10. For the polar equation in #9, find $\frac{dy}{dx}$. ~~and identify~~ the slope of the tangent lines at the origin.

1

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\cdot\sin\theta + r\cdot\cos\theta}{\frac{dr}{d\theta}\cdot\cos\theta - r\sin\theta}$$

$$\frac{dr}{d\theta} = -2\sin\theta$$

$$\frac{dy}{dx} = \frac{-2\sin\theta\cdot\sin\theta + (1+2\cos\theta)\cdot\cos\theta}{-2\sin\theta\cdot\cos\theta - (1+2\cos\theta)\cdot\sin\theta}$$

$$= \frac{-2\sin^2\theta + \cos\theta + 2\cos^2\theta}{-2\sin\theta\cos\theta - \sin\theta - 2\sin\theta\cos\theta}$$

$$= \boxed{\frac{2\cos^2\theta - 2\sin^2\theta + \cos\theta}{-4\sin\theta\cos\theta - \sin\theta}}$$

This simplified answer is enough

$$= \boxed{\frac{2\cdot\cos(2\theta) + \cos\theta}{-2\cdot\sin(2\theta) - \sin\theta}}$$

This simplified answer is fine, too.

2

Now find the slopes of the two tangent lines at the origin. The graph is at the origin when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$. [I'll use the further simplified form of dy/dx.]

$\theta = \frac{2\pi}{3}$:

$$m_T = \frac{2\cdot\cos\left(\frac{4\pi}{3}\right) + \cos\left(\frac{2\pi}{3}\right)}{-2\sin\left(\frac{4\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right)}$$

$$= \frac{2\cdot\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)}{-2\cdot\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}} = \frac{-1 + \frac{-1}{2}}{\frac{2\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{-3}{2}}{\frac{\sqrt{3}}{2}} = \frac{-3}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-3}{\sqrt{3}} = \boxed{-\sqrt{3}}$$

$\theta = \frac{4\pi}{3}$: Because of symmetry, $m_T = \boxed{+\sqrt{3}}$