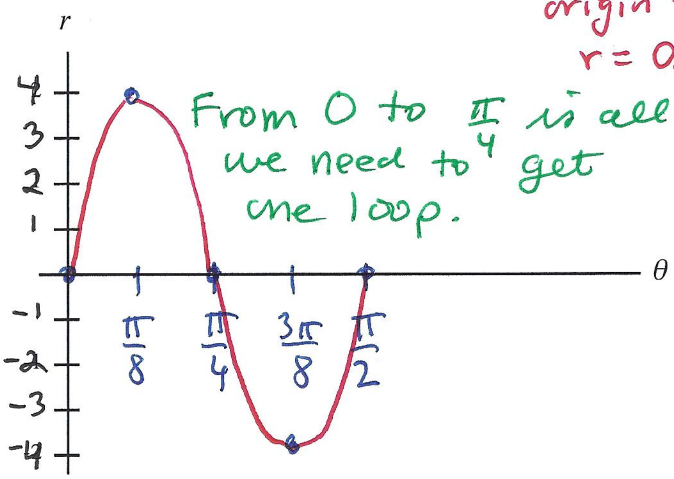
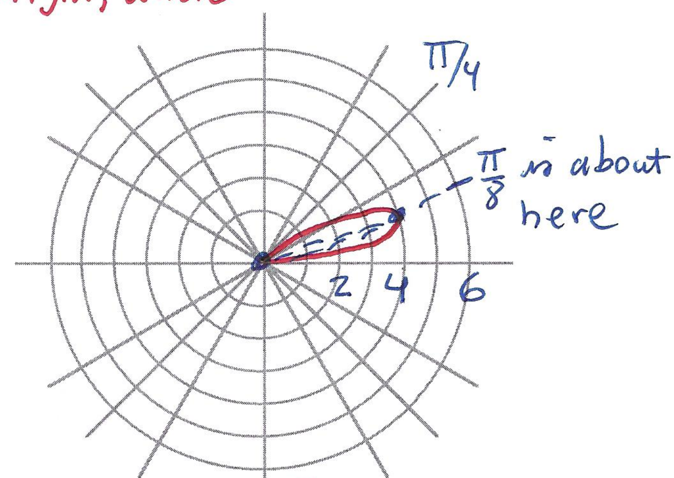
13. Find the area of the region enclosed by one loop of the curve: $r = 4\sin(4\theta)$
(Hint: Sketch enough to create one loop.) This means, from origin to origin, where $r = 0$.

$\begin{cases} \text{ampl.} = 4 \\ \text{per} = \frac{2\pi}{4} = \frac{\pi}{2} \end{cases}$





$$A_{loop} = \frac{1}{2}\int_0^{\pi/4} r^2\,d\theta \qquad \left[\sin^2 A = \tfrac{1}{2}(1-\cos(2A))\right]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} (4\sin(4\theta))^2\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} 16\cdot\sin^2(4\theta)\,d\theta \qquad A = 4\theta \text{ so } 2A = 8\theta$$

$$= \frac{1}{2}\cdot\frac{16}{1}\int_0^{\pi/4} \frac{1}{2}(1-\cos(8\theta))\,d\theta$$

$$= \frac{16}{4}\int_0^{\pi/4} [1-\cos(8\theta)]\,d\theta$$

$$= 4\left[\theta - \frac{1}{8}\sin(8\theta)\right]\Big|_0^{\pi/4}$$

$$= 4\left[\frac{\pi}{4} - \frac{1}{8}\sin(2\pi)\right] - 4\left[0 - \frac{1}{8}\cdot 0\right]$$

$$= 4\left[\frac{\pi}{4} - 0\right] - 0$$

$$= \boxed{\pi}$$

Section 10.3

14. Find the Cartesian equation for the curve defined by $r = \tan\theta\sec\theta$

we can use these identities:

1. $x = r\cdot\cos\theta$
2. $y = r\cdot\sin\theta$
3. $r = \sqrt{x^2+y^2}$
4. $\tan\theta = \frac{y}{x}$

Let's first use (4)

$$r = \frac{y}{x}\cdot\sec\theta$$

Now write $\sec\theta$ as $\frac{1}{\cos\theta}$

$$r = \frac{y}{x}\cdot\frac{1}{\cos\theta}$$

Now multiply each side by $\cos\theta$

$$r\cdot\cos\theta = \frac{y}{x}\cdot\frac{1}{\cos\theta}\cdot\cos\theta$$

Now use (1)

$$x = \frac{y}{x}\cdot 1$$

$$x^2 = y \quad \text{or} \quad \boxed{y = x^2}$$