Math 1B **Test 3 Handout**

Work (Sec. 6.4)

General Formula:

Work = Force x Distance

Hooke's Law (springs):

$$f(x) = kx,$$

where k is the spring constant and
x is the number of feet the spring is stretched *beyond its natural length*

Arc Length (8.1 & 10.2)

$$\text{Arc Length, } L = \int_a^b ds$$

$$\text{Where } \boldsymbol{ds} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ \boldsymbol{dx}$$

$$\text{or } \boldsymbol{ds} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ \boldsymbol{dy}$$

$$\text{or } \boldsymbol{ds} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ \boldsymbol{dt}$$

Surface Area (Sec. 8.2)

$$\text{Surface Area, } A = \int_a^b 2\pi r\, ds$$

Where r is the radius of revolution.

ds for surface area is the same as for arc length.

Calculus of Parametric Equations (10.2)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\text{Area, } A = \int_a^b y\frac{dx}{dt}\,dt$$

Polar Equations (10.3 & 10.4)

Rectangular relationships:

$$x = r\cos\theta \qquad y = r\sin\theta$$

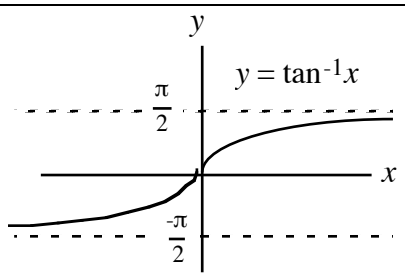
$$x^2 + y^2 = r^2 \qquad \tan\theta = \frac{y}{x}$$

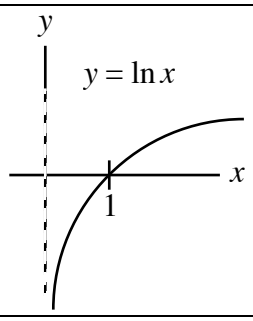
Slope of Tangent Line

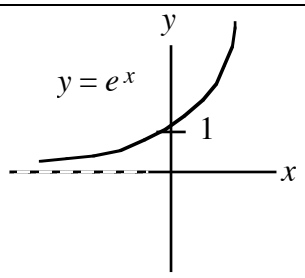
$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\cdot\sin\theta + r\cdot\cos\theta}{\frac{dr}{d\theta}\cdot\cos\theta - r\cdot\sin\theta}$$

Area of a Polar Region

$$\text{Area, } A = \int_a^b \frac{1}{2}r^2\,d\theta$$







Trigonometry

Some Trigonometric Identities:

1. $\cos^2 x + \sin^2 x = 1$

2. $\tan^2 x + 1 = \sec^2 x$

3. $\sin(2x) = 2\sin x \cos x$

4. $\cos(2x) = \cos^2 x - \sin^2 x$

5. $\cos^2 x = \frac{1}{2}\left[1 + \cos(2x)\right]$

6. $\sin^2 x = \frac{1}{2}\left[1 - \cos(2x)\right]$

Some Trigonometric Integrals:

1. $\int \tan u \, du = \ln|\sec u| + C$

2. $\int \cot u \, du = \ln|\sin u| + C$

3. $\int \sec u \, du = \ln|\sec u + \tan u| + C$

4. $\int \csc u \, du = \ln|\csc u - \cot u| + C$

5. $\int \frac{1}{u^2 + a^2} \, du = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$

6. $\int \frac{\sqrt{u^2 + a^2}}{u} \, du = \sqrt{u^2 + a^2} - a \ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

7. $\int \sqrt{u^2 + a^2} \, du = \frac{u}{2}\sqrt{u^2 + a^2} - \frac{a^2}{2} \ln\left(a + \sqrt{u^2 + a^2}\right) + C$

Quadrant I Trig Values

Degrees	0°	30°	45°	60°	90°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined