Math 1B

Chapter 11 Power Series Practice Test

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Name

1. Use the Ratio Test to determine the interval of convergence for x. Be sure to check for endpoint convergence.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(5x)^n}{n^2 4^n}$ **b)** $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n}$ **c)** $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(x-2)^n}{n+1}$

2. For a Power Series that represents a function, the Foundation Series is

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots + x^n + \ldots$$

with an interval of convergence of $|x| \le 1$ or $(-1, 1)$.

Using this Foundation power Series, find each of the following. Write the answer in both summation (sigma) notation and expanded form and identify the interval of convergence.

a) $f(x) = \dfrac{x}{1+x^2}$ b) $f(x) = \dfrac{4}{x+2}$

c) Based on your answer to part **(a)**, find the power series representation for $f(x) = \ln(1 + x^2)$

d) Based on your answer to part **(b)**, find the power series representation for $f(x) = \dfrac{1}{(x+2)^2}$

Taylor Series:

If $f(x)$ can be represented by a power series $\displaystyle\sum_{n=0}^{\infty} a_n \frac{(x-c)^n}{n!}$ with Radius of Convergence, R, then

$$f(x) = f(c) + f'(c)(x-c) + f''(c)\frac{(x-c)^2}{2!} + f'''(c)\frac{(x-c)^3}{3!} + \ldots + f^{(n)}(c)\frac{(x-c)^n}{n!} + \ldots$$

3. Find the Taylor Series representation of each function, centered at $a = 0$.

a) $f(x) = \cos(2x)$ **b)** $f(x) = x \cdot \sin x$

4. Find the Taylor Series representation of $f(x)$, centered at the given value of a.

a) $f(x) = \cos x, \quad a = \pi$ b) $f(x) = \dfrac{1}{x}, \quad a = \dfrac{1}{2}$