# Adding and Subtracting Fractions

## ADDING AND SUBTRACTING LIKE FRACTIONS

Two or more fractions are **like fractions** if they have the same denominator.

To add (or subtract) like fractions means to either add (or subtract) their numerators. The resulting fraction is *like* the original fractions and has the same denominator.

**Adding Like Fractions**:

$$\frac{a}{\boldsymbol{b}} + \frac{c}{\boldsymbol{b}} = \frac{a+c}{\boldsymbol{b}}$$

*Example:*

$$\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$$

**Subtracting Like Fractions**:

$$\frac{a}{\boldsymbol{d}} - \frac{c}{\boldsymbol{d}} = \frac{a-c}{\boldsymbol{d}}$$

*Example:*

$$\frac{6}{7} - \frac{4}{7} = \frac{6-4}{7} = \frac{2}{7}$$

**Caution:** Notice that the denominators are the same throughout each example. When adding or subtracting fractions, we do not perform any operation on the denominators.

**Group Exercise 1** Combine each pair of like fractions (add or subtract as indicated). Simplify the result if possible.

a) $\frac{7}{15} + \frac{1}{15}$

b) $\frac{10}{13} - \frac{5}{13}$

c) $\frac{11}{12} + \frac{5}{12}$

d) $\frac{7}{10} - \frac{1}{10}$

## COMBINING FRACTIONS CONTAINING VARIABLES

Two or more terms are considered to be ***like terms*** if they have exactly the same variable and exponent, even if the coefficients are different. This is true even when the coefficient is a fraction.

To combine like terms we add their coefficients. For example, $2x$ and $3x$ are like terms, and $2x + 3x = 5x$.

Similarly, $\frac{2}{7}x$ and $\frac{3}{7}x$ are like terms and $\frac{2}{7}x + \frac{3}{7}x = \frac{5}{7}x$.

This could also be written as $\frac{2x}{7} + \frac{3x}{7} = \frac{5x}{7}$.

**Group Exercise 2** Combine these fractions. Simplify the result, if possible.

a) $\frac{4}{9}x + \frac{8}{9}x$

b) $\frac{1}{14}w - \frac{-9}{14}w$

c) $-\frac{17}{30}b + \frac{11}{30}b$

d) $-\frac{7}{20}q - \frac{13}{20}q$

## USING THE LCD TO ADD AND SUBTRACT UNLIKE FRACTIONS

Fractions with different denominators cannot be added directly. For example, $\frac{1}{6}$ and $\frac{2}{15}$ cannot be added until we rewrite them with a common denominator, (**CD**). In other words, we must build up each fraction to have the same denominator, and there are many options:

$\frac{1}{6}$ and $\frac{2}{15}$ can both be built up to have a common denominator (CD) of 30, of 60, of 90, and so on:

| CD = **30** | CD = **60** | CD = **90** |
|---|---|---|
| $\frac{1}{6} \cdot \frac{5}{5} = \frac{5}{\mathbf{30}}$ | $\frac{1}{6} \cdot \frac{10}{10} = \frac{10}{\mathbf{60}}$ | $\frac{1}{6} \cdot \frac{15}{15} = \frac{15}{\mathbf{90}}$ |
| $\frac{2}{15} \cdot \frac{2}{2} = \frac{4}{\mathbf{30}}$ | $\frac{2}{15} \cdot \frac{4}{4} = \frac{8}{\mathbf{60}}$ | $\frac{2}{15} \cdot \frac{6}{6} = \frac{12}{\mathbf{90}}$ |

The most efficient CD is 30, called the *least common denominator* (**LCD**), but all of these will work fine. Just remember to simply all results.

**Example 1:** Evaluate each by first finding the LCD. Simplify the result.

a) $\frac{1}{6} + \frac{2}{15}$ b) $\frac{7}{15} - \frac{4}{5}$ .

**Procedure:** Build up each fraction to have the given LCD.

**Answer:**

a) LCD = 30

$$\frac{1}{6} \cdot \mathbf{\frac{5}{5}} + \frac{2}{15} \cdot \mathbf{\frac{2}{2}}$$

$$= \frac{5}{30} + \frac{4}{30}$$

$$= \frac{9}{30}$$ ← This simplifies by a factor of 3.

$$= \frac{3}{10}$$

b) LCD = 15

$$\frac{7}{15} - \frac{4}{5} \cdot \mathbf{\frac{3}{3}}$$

$$= \frac{7}{15} - \frac{12}{15}$$

$$= \frac{-5}{15}$$ ← This can be simplified by a factor of 5.

$$= \frac{-1}{3}$$

**Group Exercise 3** Evaluate each sum or difference by first finding the LCD and building up the fractions appropriately. Be sure to simplify each answer.

a) $\frac{7}{10} + \frac{1}{8}$ b) $\frac{9}{10} - \frac{4}{15}$ c) $\frac{2}{5} - \frac{3}{4}$

## COMBINING UNLIKE FRACTIONS WITH VARIABLES

If the coefficients of two like terms have different denominators, then we cannot add (or subtract) them until the fractions have the same denominator.

It might be helpful to write the variable in the numerator before getting the common denominator. This is demonstrated in Example 2.

**Example 4:** Combine these like terms.

$$\frac{1}{12}x + \frac{7}{15}x$$

**Procedure:** First write the variable in each numerator. The LCD is 60. Simplify the result.

**Answer:**

$$\frac{1x}{12} + \frac{7x}{15}$$

$$= \frac{1x}{12} \cdot \mathbf{\frac{5}{5}} + \frac{7x}{15} \cdot \mathbf{\frac{4}{4}}$$

$$= \frac{5x}{60} + \frac{28x}{60}$$

$$= \frac{33x}{60}$$ ← This simplifies by a factor of 3.

$$= \frac{11x}{20} \quad \text{or} \quad \frac{11}{20}x$$

**Group Exercise 4** Combine these like terms. Use Example 4 as a guide.

a) $\frac{4}{9}b + \frac{7}{18}b$

b) $-\frac{11}{8}w + \frac{1}{4}w$

c) $-\frac{7}{8}y - \frac{3}{10}y$

# Focus Exercises

*Add or subtract as indicated. Simplify the results, if possible.*

**1.** $\frac{4}{9}c + \frac{2}{9}c$

**2.** $\frac{1}{12}k + \frac{5}{12}k$

**3.** $\frac{17}{20}q - \frac{9}{20}q$

**4.** $\frac{7}{30}x - \frac{19}{30}x$

**5.** $\frac{3}{10}y - \frac{7}{8}y$

**6.** $\frac{2}{9}p - \frac{11}{12}p$

**7.** $-\frac{8}{15}d + \frac{7}{10}d$

**8.** $-\frac{9}{10}m - \frac{3}{4}m$