Complex Fractions, Part 1

A **complex fraction** is any fraction in which the numerator or the denominator, or both, contain one or more fractions. Examples of complex fractions include

$$\frac{5}{\frac{7}{8}} \qquad \frac{\frac{2}{3}}{6} \qquad \frac{1}{\frac{5}{2}} \qquad \frac{\frac{x}{2y}}{\frac{3}{y^2}} \qquad \frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}} \qquad \frac{\frac{1}{x}-\frac{3}{2x}}{1+\frac{5}{x^2}}$$

SIMPLIFYING COMPLEX FRACTIONS, METHOD 1

A fraction bar is also a division bar, so we can treat, for example, the complex fraction $\frac{\frac{2}{3}}{\frac{5}{4}}$ as one fraction divided by another:

A complex fraction containing *no addition or subtraction* can be simplified by first writing the fraction as division.	For example, $\frac{\frac{2}{3}}{\frac{5}{4}}$ can be written as $\frac{2}{3} \div \frac{\mathbf{5}}{\mathbf{4}}$.
Simplify by multiplying the first fraction by the reciprocal of the second fraction and multiplying:	and $\frac{2}{3} \div \frac{\mathbf{5}}{\mathbf{4}} = \frac{2}{3} \cdot \frac{\mathbf{4}}{\mathbf{5}} = \frac{8}{15}$,

Rule of Dividing Fractions: ***Multiply the Reciprocal***

Change Division to multiplication by writing the first fraction as it is and the reciprocal of the second fraction:

$$\frac{a}{b} \div \frac{\boldsymbol{c}}{\boldsymbol{d}} = \frac{a}{b} \cdot \frac{\boldsymbol{d}}{\boldsymbol{c}}$$

Group Exercise 1 Simplify each complex fraction by first writing it using the division symbol. Compare answers with members in your group.

a) $\frac{\frac{4}{7}}{\frac{5}{3}}$

b) $\frac{\frac{m}{p}}{\frac{2p}{t}}$

Group Exercise 2 Simplify each complex fraction in the first column by first writing it using the division symbol. All the steps in part a) are shown. Compare answers with members in your group.

	Complex Fraction	Notes:	Written as Division	Written as Multiplication	Simplified Result
a)	$\dfrac{5}{\frac{7}{8}}$	The numerator is the whole number 5; this can be written as a fraction with 1 in the denominator: $\frac{5}{1}$.	$= 5 \div \dfrac{7}{8}$	$= \dfrac{5}{1} \cdot \dfrac{8}{7}$	$= \dfrac{40}{7}$
b)	$\dfrac{w}{\frac{x}{w}}$	The numerator, w, can be written as a fraction with 1 in the denominator: $\frac{w}{1}$.			
c)	$\dfrac{\frac{2}{3}}{6}$	This time it is the denominator, 6, that can be written as a fraction with 1 in the denominator: $\frac{6}{1}$.			
d)	$\dfrac{\frac{2}{3}}{\frac{8}{9}}$	Here, both the numerator and denominator are full fractions. Be sure to simplify the results.			
e)	$\dfrac{\frac{x}{2y}}{\frac{3}{y^2}}$	Be sure to simplify the results.			

SIMPLIFYING COMPLEX FRACTIONS, METHOD 2

A second method of simplifying complex fractions is to clear all denominators directly while in its complex form. We do so by multiplying the whole fractions by a carefully chosen value of 1, such as

$$\frac{7}{7}\ ,\ \frac{x}{x}\ ,\text{ or }\ \frac{3w^2}{3w^2}\ .$$

For example, in the complex fraction $\dfrac{\frac{4}{5}}{\frac{3}{2}}$, we can clear the denominators, 2 and 5, by using the common multiplier of 10, multiplying the whole fraction by $\dfrac{10}{10}$. (10 is the least common denominator for $\frac{4}{5}$ and $\frac{3}{2}$.)

In this case, $\frac{10}{10}$ is better written as $\dfrac{\frac{10}{1}}{\frac{10}{1}}$ so that we can more easily multiply:

$$\frac{\frac{4}{5}}{\frac{3}{2}} \cdot \frac{\frac{10}{1}}{\frac{10}{1}} = \frac{\frac{4}{5} \cdot \frac{10}{1}}{\frac{3}{2} \cdot \frac{10}{1}} = \frac{\frac{40}{5}}{\frac{30}{2}} = \frac{8}{15}$$

Note: We use **LCD** to abbreviate *least common denominator*.

Let's put Method 2 into practice.

Example 1: Simplify each complex fraction using Method 2.

a) $\dfrac{3}{\frac{7}{8}}$ b) $\dfrac{\frac{7}{8}}{\frac{3}{4}}$ c) $\dfrac{\frac{3}{x}}{\frac{2}{x^2}}$ d) $\dfrac{\frac{x}{2y}}{\frac{3}{y^2}}$

Procedure: First recognize the least common denominator (LCD), then use it as the common multiplier. In part a), write the numerator as a fraction, $\frac{3}{1}$. Follow each step carefully.

Answer:

a) LCD = 8: $\dfrac{\frac{3}{1}}{\frac{7}{8}} \cdot \dfrac{\frac{8}{1}}{\frac{8}{1}} = \dfrac{\frac{3}{1} \cdot \frac{8}{1}}{\frac{7}{8} \cdot \frac{8}{1}} = \dfrac{\frac{3}{1} \cdot \frac{8}{1}}{\frac{7}{1} \cdot \frac{1}{1}} = \dfrac{40}{7}$

b) LCD = 8: $\dfrac{\frac{7}{8}}{\frac{3}{4}} \cdot \dfrac{\frac{8}{1}}{\frac{8}{1}} = \dfrac{\frac{7}{8} \cdot \frac{8}{1}}{\frac{3}{4} \cdot \frac{8}{1}} = \dfrac{\frac{7}{1} \cdot \frac{1}{1}}{\frac{3}{1} \cdot \frac{2}{1}} = \dfrac{7}{6}$

c) LCD = x^2: $\dfrac{\frac{3}{x}}{\frac{2}{x^2}} \cdot \dfrac{\frac{x^2}{1}}{\frac{x^2}{1}} = \dfrac{\frac{3}{x} \cdot \frac{x^2}{1}}{\frac{2}{x^2} \cdot \frac{x^2}{1}} = \dfrac{\frac{3}{1} \cdot \frac{x}{1}}{\frac{2}{1} \cdot \frac{1}{1}} = \dfrac{3x}{2}$

d) LCD = $2y^2$: $\dfrac{\frac{x}{2y}}{\frac{3}{y^2}} \cdot \dfrac{\frac{2y^2}{1}}{\frac{2y^2}{1}} = \dfrac{\frac{x}{2y} \cdot \frac{2y^2}{1}}{\frac{3}{y^2} \cdot \frac{2y^2}{1}} = \dfrac{\frac{x}{1} \cdot \frac{y}{1}}{\frac{3}{1} \cdot \frac{2}{1}} = \dfrac{xy}{6}$

Group Exercise 3 Simplify each complex fraction using Method 2. Compare answers with members in your group.

a) $\dfrac{\frac{5}{8}}{\frac{3}{2}}$

b) $\dfrac{\frac{4}{9}}{\frac{8}{3}}$

c) $\dfrac{\frac{4y^2}{w^2}}{\frac{8y}{3w}}$

Focus Exercises

Simplify each complex fraction using <u>any method</u>.

1. $$\frac{\frac{5}{3}}{\frac{7}{4}}$$

2. $$\frac{\frac{10}{9}}{\frac{25}{12}}$$

3. $$\frac{\frac{4}{11}}{8}$$

4. $$\frac{12}{\frac{6}{5}}$$

5. $$\frac{\frac{3m}{p}}{\frac{5m}{2p}}$$

6. $$\frac{\frac{u}{3v}}{6u}$$

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c)	$\dfrac{\frac{2}{3}}{6}$	This time it is the denominator, 6, that can be written as a fraction with 1 in the denominator: $\frac{6}{1}$.	$= \dfrac{2}{3} \div \dfrac{6}{1}$	$= \dfrac{2}{3} \cdot \dfrac{1}{6}$	$= \dfrac{2}{18} = \dfrac{1}{9}$
d)	$\dfrac{\frac{x}{2y}}{\frac{3}{y^2}}$	Be sure to simplify the results.	$= \dfrac{x}{2y} \div \dfrac{3}{y^2}$	$= \dfrac{x}{2y} \cdot \dfrac{y^2}{3}$	$= \dfrac{xy^2}{6y} = \dfrac{xy}{6}$