Complex Fractions, Part 2

As we have seen previously, a **complex fraction** is any fraction in which the numerator or the denominator contain one or more fractions. Examples of complex fractions include ...

$$\frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}} \qquad \frac{\frac{1}{x}-\frac{3}{2x}}{1+\frac{5}{x^2}}$$

We simplify a complex fraction contain addition and/or subtraction using multiplying the complex fraction by the LCD of *all* of the smaller fractions within.

For example, the complex fraction $\frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}}$ has four denominators within it: 3, 6, 4, and 2. The LCD is **12**, so the common multiplier for the entire fraction is 12, and we use it to clear each of the denominators within the complex fraction.

Multiply the whole fraction by $\frac{12}{12}$, better written as $\frac{\frac{12}{1}}{\frac{12}{1}}$, as shown here:

$$= \frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}} \cdot \frac{\frac{\mathbf{12}}{\mathbf{1}}}{\frac{\mathbf{12}}{\mathbf{1}}}$$

Multiply $\frac{12}{1}$ to both the numerator and the denominator.

Distribute $\frac{12}{1}$ to each term (fraction) in the numerator and to each term in the denominator.

$$= \frac{\frac{2}{3}\cdot\frac{\mathbf{12}}{\mathbf{1}} + \frac{5}{6}\cdot\frac{\mathbf{12}}{\mathbf{1}}}{\frac{5}{4}\cdot\frac{\mathbf{12}}{\mathbf{1}} - \frac{1}{2}\cdot\frac{\mathbf{12}}{\mathbf{1}}}$$

Simplify each product of fractions within by dividing out common factors. If done correctly, all denominators in the complex fractions will divide out (cancel out).

$$= \frac{8+10}{15-6}$$

Simplify in the numerator and in the denominator. Simplify completely.

$$= \frac{18}{9} = \boxed{2}$$

Your work might look a little more like this:

$$\frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}} \cdot \frac{\frac{12}{1}}{\frac{12}{1}} = \frac{\frac{2}{{}_1\cancel{3}}\cdot\frac{\cancel{12}^{4}}{1} + \frac{5}{{}_1\cancel{6}}\cdot\frac{\cancel{12}^{2}}{1}}{\frac{5}{{}_1\cancel{4}}\cdot\frac{\cancel{12}^{3}}{1} - \frac{1}{{}_1\cancel{2}}\cdot\frac{\cancel{12}^{6}}{1}} = \frac{8+10}{15-6} = \frac{18}{9} = 2$$

... or you might distribute and cancel mentally as you go, such as ...

$$\frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}} \cdot \frac{\frac{\mathbf{12}}{\mathbf{1}}}{\frac{\mathbf{12}}{\mathbf{1}}} = \frac{8+10}{15-6} = \frac{18}{9} = 2$$

Group Exercise 1 Simplify each complex fraction.

a) $$\frac{\frac{5}{12} + \frac{1}{3}}{\frac{2}{3} - \frac{1}{4}}$$

b) $$\frac{\frac{3}{8} + \frac{1}{6}}{2 - \frac{1}{4}}$$

Example: Simplify each higher level complex fraction by using Method 2, described above.

a) $$\frac{\frac{4}{x} - \frac{3}{2x}}{1 + \frac{5}{x^2}}$$ b) $$\frac{\frac{1}{3} - \frac{1}{3w} - \frac{4}{w^2}}{\frac{1}{3} - \frac{3}{w^2}}$$

Procedure: Make every term a fraction, if it isn't already. That will help in the multiplication process. Next, find the least common denominator and use it to create a form of 1.

Answer: a) First, the common denominator is $2x^2$. Let's go right to the distribution step where each small fraction is multiplied by the $\frac{2x^2}{1}$

$$\frac{\frac{4}{x}\cdot\frac{2x^2}{1} - \frac{3}{2x}\cdot\frac{2x^2}{1}}{\frac{1}{1}\cdot\frac{2x^2}{1} + \frac{5}{x^2}\cdot\frac{2x^2}{1}} = \frac{\frac{4}{1}\cdot\frac{2x}{1} - \frac{3}{1}\cdot\frac{x}{1}}{\frac{1}{1}\cdot\frac{2x^2}{1} + \frac{5}{1}\cdot\frac{2}{1}} = \frac{8x - 3x}{2x^2 + 10} = \frac{5x}{2x^2 + 10}$$

This fraction cannot simplify any further.

b) The common denominator is $3w^2$. Let's distribute $\frac{3w^2}{1}$ to every small fraction.

$$\frac{\frac{1}{3}\cdot\frac{3w^2}{1} - \frac{1}{3w}\cdot\frac{3w^2}{1} - \frac{4}{w^2}\cdot\frac{3w^2}{1}}{\frac{1}{3}\cdot\frac{3w^2}{1} - \frac{3}{w^2}\cdot\frac{3w^2}{1}} = \frac{\frac{1}{1}\cdot\frac{w^2}{1} - \frac{1}{1}\cdot\frac{w}{1} - \frac{4}{1}\cdot\frac{3}{1}}{\frac{1}{1}\cdot\frac{w^2}{1} - \frac{3}{1}\cdot\frac{3}{1}}$$

$$= \frac{w^2 - w - 12}{w^2 - 9} = \frac{(w + 3)(w - 4)}{(w - 3)(w + 3)} = \frac{(w - 4)}{(w - 3)}$$

This fraction *can* simplify further. ↑ Factor it and divide out the common factor.

Group Exercise 2 Simplify each complex fraction.

a) $\dfrac{\frac{v}{4} - \frac{1}{v}}{\frac{1}{2} - \frac{1}{v}}$

b) $\dfrac{\frac{v}{4} - \frac{1}{v}}{\frac{1}{2} - \frac{1}{v}}$

c) $\dfrac{\frac{1}{x} + \frac{3}{x^2}}{1 - \frac{9}{x^2}}$

d) $\dfrac{\frac{1}{2} - \frac{3}{x} + \frac{4}{x^2}}{1 - \frac{2}{x}}$

Focus Exercises

Simplify each complex fraction using Method 2.

1. $$\frac{1 - \frac{5}{2}}{1 + \frac{2}{3}}$$

2. $$\frac{\frac{5}{12} + \frac{1}{3}}{\frac{2}{3} - \frac{1}{4}}$$

3. $$\frac{\frac{2}{3} - \frac{1}{2}}{1 - \frac{1}{6}}$$

4. $$\frac{1 + \frac{2}{y}}{\frac{y}{3} + \frac{2}{3}}$$

5. $$\frac{\frac{1}{6} - \frac{1}{2w}}{\frac{1}{3w} - \frac{1}{w^2}}$$

6. $$\frac{1 - \frac{1}{y^2}}{\frac{1}{y} + \frac{1}{y^2}}$$

7. $$\frac{\frac{1}{6} + \frac{4}{3x} + \frac{2}{x^2}}{1 + \frac{2}{x}}$$

8. $$\frac{\frac{1}{2} - \frac{3}{x} + \frac{4}{x^2}}{1 - \frac{2}{x}}$$