Composite Functions

FUNCTIONAL VALUES

In a previous discussion, we looked at finding values of a function by replacing the **argument** with a number (in the domain).

For example, for $f(x) = -5x + 4$, we can replace the argument, $\boldsymbol{x}$, with 3 or -3 and find the corresponding range value:

a) $f(3) = -5(3) + 4$

$= -15 + 4$

$= -11$

This gives the ordered pair (3, -11).

b) $f(-3) = -5(-3) + 4$

$= 15 + 4$

$= 19$

This gives the ordered pair (-3, 19).

We can also replace the argument with other variable arguments.

For example, for $f(x) = -5x + 4$, we can replace the argument, x, with a, with $2a$, or with $a + 3$:

a) $f(a) = -5(a) + 4$ $= -5a + 4$	b) $f(2a) = -5(2a) + 4$ $= -10a + 4$	c) $f(a+3) = -5(a+3) + 4$ $= -5a - 15 + 4$ $= -5a - 11$

Note: We don't typically make ordered pairs from these types of values.

As a reminder, the new argument gets replaced in each and every x-value.

Example 1: Given $g(x) = x^2 - 3x - 4$, find the following:

a) $g(a)$ b) $g(2a)$ c) $g(a+1)$

Procedure: For each, replace x with the requested argument.

Answer:

a) Replace each x with a:

$$g(a) = a^2 - 3a - 4$$

b) Replace each x with $2a$; simplify.

$g(2a) = (2a)^2 - 3(2a) - 4$

$$g(2a) = 4a^2 - 6a - 4$$

c) Replace each x with the quantity $(a + 1)$:

$g(a+1) = (a+1)^2 - 3(a+1) - 4$

$= a^2 + 2a + 1 - 3a - 3 - 4$

Simplify by combining like terms

$$g(a+1) = a^2 - a - 6$$

Group Exercise 1 Given $h(x) = \dfrac{x^2 - 4}{x - 1}$, find the following:

a) $h(3a)$

b) $h(a - 2)$

It's also possible the new argument contains an x.

Group Exercise 2 Given $h(x) = 9 - 2x$, find the following:

a) $h\left(\frac{1}{2}x\right)$

b) $h(x^2)$

c) $h(5x + 1)$

COMPOSITE FUNCTIONS

In fact, we can also replace an argument in one function, *f(x)*, with another function, *g(x)*. This is called the ***composition*** of two functions. The result is another function.

> *f(x)* composed with *g(x)* is written $f \circ g\,(x)$, which can also be written as $f[g(x)]$
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It is even possible to consider a function composed with itself: $f \circ f\,(x) = f[f(x)]$. Crazy!

Example 3: Given $f(x) = 3x - 1$ and $g(x) = x^2 + 2$, find the following:

a) $f \circ g\,(x)$

b) $g \circ f\,(x)$

c) $f \circ f\,(x)$

Procedure: First write each with the second function "inside" the first. The "replacement value" is a full function.

Answer:

a) $f \circ g\,(x)$

$$\begin{aligned} f[g(x)] &= f(x^2 + 2) \\ &= 3(x^2 + 2) - 1 \\ &= 3x^2 + 6 - 1 \\ &= 3x^2 + 5 \end{aligned}$$

b) $g \circ f\,(x)$

$$\begin{aligned} g\,[f(x)] &= g(3x - 1) \\ &= (3x - 1)^2 + 2 \\ &= 9x^2 - 6x + 1 + 2 \\ &= 9x^2 - 6x + 3 \end{aligned}$$

c) $f \circ f\,(x)$

$$\begin{aligned} f[g(x)] &= f(3x - 1) \\ &= 3(3x - 1) - 1 \\ &= 9x - 3 - 1 \\ &= 9x - 4 \end{aligned}$$

Group Exercise 3 Given $f(x) = 3x - 2$ and $g(x) = x^2 - x$, find the following:

a) $f \circ g\,(x)$

b) $g \circ f\,(x)$

Group Exercise 4 Given $f(x) = \frac{2}{3}x - 1$ and $g(x) = \frac{3x + 3}{2}$, find the following:

a) $f \circ g\,(x)$

b) $g \circ f\,(x)$

Focus Exercises

Given $f(x) = 2x - 3$ and $g(x) = x^2 - x + 2$, find the following.

1. $f(3w)$

2. $f(2c^2)$

3. $f(x - 2)$

4. $g(3w)$

5. $g(2c^2)$

6. $g(x - 2)$

7. $f \circ g\,(x)$

8. $f \circ f\,(x)$

9. $g \circ f\,(x)$