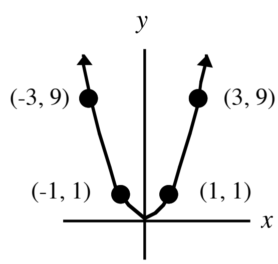
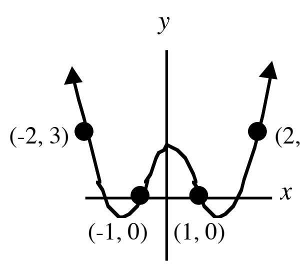
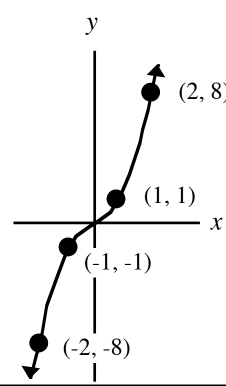
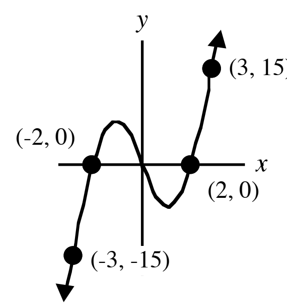
Even & Odd Functions

Some functions—*not all*—have the property of being either "odd" or "even" based on these criteria:

Function Type	Graph Symmetry	Algebraic Property
Even	about the ***y*-axis**	$f(-x) = f(x)$
Odd	about the **origin**	$f(-x) = -f(x)$

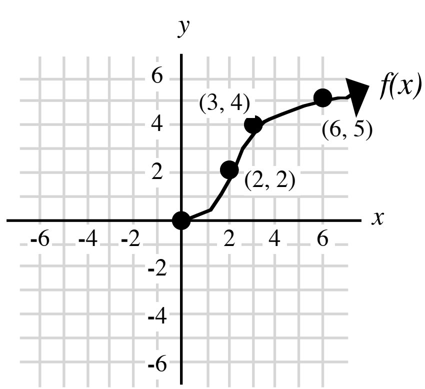
Her are examples of the graphs of even and odd functions:

Even	**Even**	**Odd**	**Odd**
$f(x) = x^2$	$g(x) = x^4 - 4x^2 + 3$	$h(x) = x^3$	$k(x) = x^3 - 4x$
			

Group Exercise 1

Given half of the graph of a function—and whether the function is odd or even—draw in the other half. Also, below each graph, fill in each blank and answer the question.

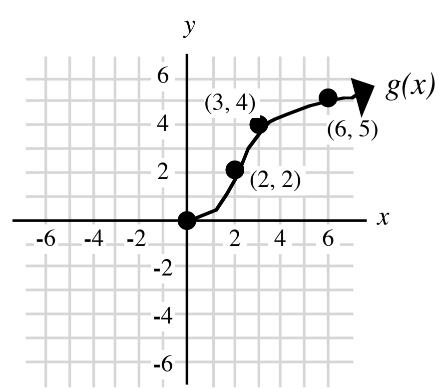
a) *f(x)* is an even function:



$f(3)$ = _______ and $f(-3)$ = _______

What can you say about $f(3)$ and $f(-3)$?

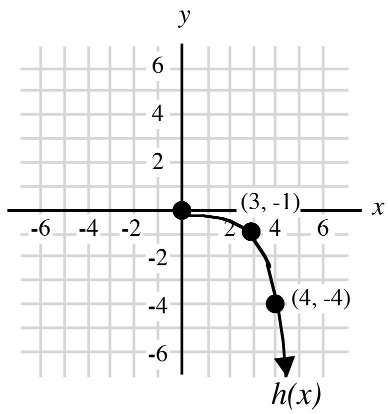
b) *g(x)* is an odd function:



$g(3)$ = _______ and $g(-3)$ = _______

What can you say about $g(3)$ and $g(-3)$?

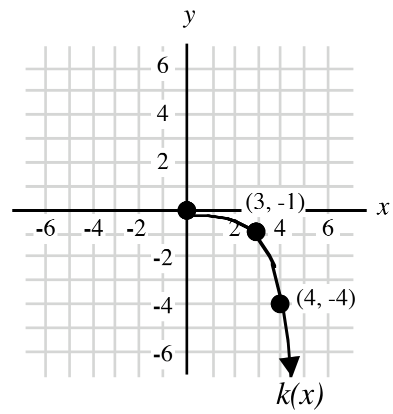
c) *h(x)* is an odd function:



$h(3)$ = _______ and $h(-3)$ = _______

What can you say about $h(3)$ and $h(-3)$?

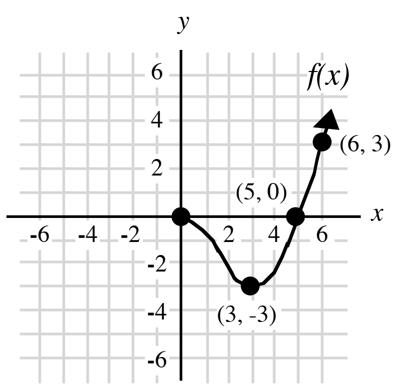
d) *k(x)* is an even function:



$k(3)$ = _______ and $k(-3)$ = _______

What can you say about $k(3)$ and $k(-3)$?

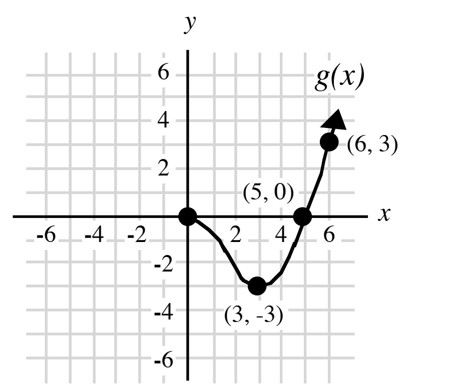
e) *f(x)* is an even function:



$f(6)$ = _______ and $f(-6)$ = _______

What can you say about $f(6)$ and $f(-6)$?

f) *g(x)* is an odd function:

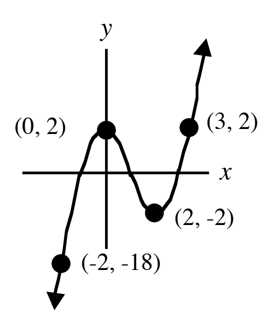


$g(6)$ = _______ and $g(-6)$ = _______

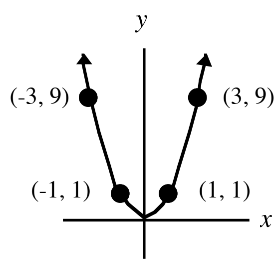
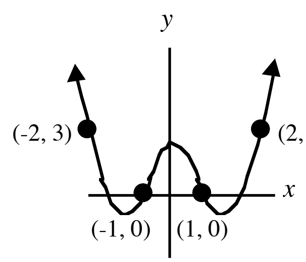
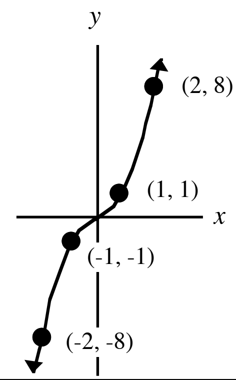
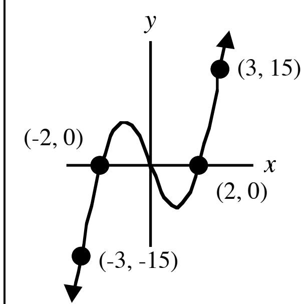
What can you say about $g(6)$ and $g(-6)$?

Group Exercise 2

Based on its graph, is this function odd or even?
Explain your answer.



Here again are the odd and even functions from the introduction:

Even	Even	Odd	Odd
$f(x) = x^2$	$g(x) = x^4 - 4x^2 + 3$	$h(x) = x^3$	$k(x) = x^3 - 4x$
			

Note 1: *g(x)* can be written $g(x) = x^4 - 4x^2 + 3x^0$.

Note 2: *k(x)* can be written $k(x) = x^3 - 4x^1$.

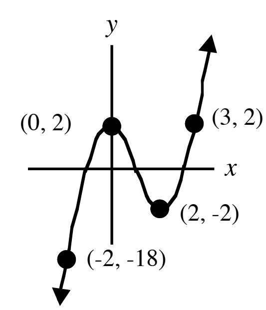
Group Exercise 3 Use the graphs above to answer these questions:

a) What do you notice about the powers in the two even functions, *f(x)* and *g(x)*?

b) What do you notice about the powers in the two odd functions, *h(x)* and *k(x)*?

Group Exercise 4 Consider the function, $p(x) = x^3 - 3x^2 + 2$, which has the graph shown at right.

Based on the function, itself, is *p(x)* odd or even or both? Explain your answer.



Group Exercise 5 What type of function is it that has symmetry about the *x*-axis? Explain your answer.

Here are the *algebraic* properties of even and odd functions:

EVEN:	A function, $f(x)$, is **even** if $f(-x) = f(x)$ for all domain values.
ODD:	A function, $f(x)$, is **odd** if $f(-x) = -f(x)$ for all domain values.

Example 1: Determine algebraically whether the function is even, odd, or neither.

a) $f(x) = x^3 - 4x$ b) $g(x) = 2x^2 + 6x - 8$ c) $h(x) = x^4 - 4x^2 + 3$

Procedure: Find $f(-x)$ and simplify. If the resulting function is the same as $f(x)$, then the function is *even*; if it is the opposite of $f(x)$, then it is *odd*; it might also be *neither* of these options.

Answer: a)

$f(x) = x^3 - 4x$	Replace each x in the function with $-x$ and simplify.
$f(-x) = (-x)^3 - 4(-x)$	Evaluate each term.
$= -x^3 + 4x$	Because the lead term is negative, factor out -1.
$= -1(x^3 - 4x)$	In the parentheses is the original $f(x)$, so ...
$f(-x) = -f(x)$	$f(x)$ is an odd function.

b)

$g(x) = 2x^2 + 6x - 8$	Replace each x in the function with $-x$ and simplify.
$g(-x) = 2(-x)^2 + 6(-x) - 8$	Evaluate each term.
$= 2x^2 - 6x - 8$	The lead term is the same as it is for $g(x)$, so g is not odd; however, the middle terms of $g(x)$ and $g(-x)$ are different, so ...
$g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$ so ...	$g(x)$ is neither even nor odd.

c)

$h(x) = x^4 - 4x^2 + 3$	Replace each x in the function with $-x$ and simplify.
$h(-x) = (-x)^4 - 4(-x)^2 + 3$	A negative value to an even power is the same as the positive value to an even power. In other words, $(-x)^2 = x^2$
$h(-x) = x^4 - 4x^2 + 3$	This is identical to the original function, so ...
$h(-x) = h(x)$	$h(x)$ is an even function.

Focus Exercises

For each, use the **algebraic properties** to determine whether the function is even, odd, or neither.

1. $f(x) = \frac{-2}{3}x$

2. $g(x) = 3x^2 - 5$

3. $h(x) = \sqrt[3]{x}$

4. $f(x) = x^3 + x + 1$

5. $g(x) = 4 - x^2$

6. $h(x) = 5 - x^3$