Factoring out the GCF and Factoring by Grouping

Factoring out the Greatest Common Factor (GCF)

To **factor a polynomial**, we first identify all of the factors common to each term, the **greatest common factor**, abbreviated **GCF**. For example,

a) in the polynomial $12x + 3$, each term has a factor of 3.

b) in the polynomial $4y^3 - 15y^2$ each term has a factor of y^2.

c) in the polynomial $10x^3 + 15x^2 - 5x$, each term has a factor of $5x$.

d) in the polynomial $7y^2 + 2$, there is no common factor other than 1.

Once we identify the GCF, we *extract* or *factor out* the GCF from each term. This creates a factored form of the polynomial; one of the factors is the GCF and the other factor is a quantity, often a binomial or a trinomial.

Note: The quantity factor is in parentheses and has the same number of terms as the original polynomial.

For example, for each polynomial, extract the GCF and write it in factored form.

a) $12x + 3$ — Extract a common factor of 3 from each term.

$= 3(4x + 1)$

← Each is in factored form →

b) $4y^3 - 15y^2$ — Extract a common factor of y^2 from each term.

$= y^2(4y - 15)$

c) $10x^3 + 15x^2 - 5x$ — Extract a common factor of $5x$ from each term.

$= 5x(2x^2 + 3x - 1)$

← Each is in factored form →

c) $7y^2 + 2$ — The GCF is 1 so there is nothing to extract, but it can still be written in factored form.

$= 1(7y^2 + 2)$

Notice that:

1. the GCF is one of the factors in the factored form, and
2. when one of the terms is the GCF, itself—as in examples a) and c)—a factor of 1 or -1 holds its place in the quantity, and
3. we can verify the factoring is accurate by distributing.

Group Exercise 1 Use distribution to verify that the factoring shown above in examples a), b), c), and d) are accurate

Group Exercise 2 Factor each polynomial by identifying and extracting the GCF.

a) $6x^2 + 9x$

b) $10y^2 - 9$

c) $6y^4 - 8y^3 + 2y^2$

d) $12x^3 - 15x^2 + x$

A Negative Lead Term

If the lead term (first term) in a polynomial is negative, extract the negative along with the GCF.

Caution: It is easy to make a mistake in signs (plus or minus) when factoring out a negative, so it is especially helpful to verify the factored form by distributing.

For example,

a) $-6x - 21$ Extract a common factor of -3 from each term.

$= -3(2x + 7)$

Verify by distributing.

b) $-5x^2 + 15x$ Extract a common factor of $-5x$ from each term.

$= -5x(x - 3)$ Notice the second term of the binomial factor is negative.

Verify by distributing.

Group Exercise 3 Factor each polynomial by identifying and extracting the GCF. Verify the answer by distribution.

a) $-3x^2 - 15x$

b) $-10y^3 + 5y^2$

c) $-2y^3 - 6y^2 + 2y$

Factoring Quadrinomials: Factor by Grouping

1. A common factor can be factored out ...

to the **left** side or to the **right** side

$\boldsymbol{a}x + \boldsymbol{a}b$

$= \boldsymbol{a}(x + b)$

$\boldsymbol{a}x + \boldsymbol{a}b$

$= (x + b)\boldsymbol{a}$

2. A common factor can be a binomial, represented here by $(\boldsymbol{B})$:

to the **left** side	or	to the **right** side
$(\boldsymbol{B}) \cdot 2x + (\boldsymbol{B}) \cdot 5$		$2x \cdot (\boldsymbol{B}) + 5 \cdot (\boldsymbol{B})$
$= (\boldsymbol{B})(2x + 5)$		$= (2x + 5)(\boldsymbol{B})$

3. Here is the same factoring exercise using an actual common binomial factor:

to the **left** side	or	to the **right** side
$(\mathbf{3}\boldsymbol{x} - \mathbf{4}) \cdot 2x + (\mathbf{3}\boldsymbol{x} - \mathbf{4}) \cdot 5$		$2x \cdot (\mathbf{3}\boldsymbol{x} - \mathbf{4}) + 5 \cdot (\mathbf{3}\boldsymbol{x} - \mathbf{4})$
$= (\mathbf{3}\boldsymbol{x} - \mathbf{4})(2x + 5)$		$= (2x + 5)(\mathbf{3}\boldsymbol{x} - \mathbf{4})$

Group Exercise 4 Factor out the common binomial factor.

a) $3x^2(\boldsymbol{x} - \mathbf{5}) + 2(\boldsymbol{x} - \mathbf{5})$

b) $2y(\boldsymbol{y}^{\mathbf{2}} + \mathbf{3}) - 7(\boldsymbol{y}^{\mathbf{2}} + \mathbf{3})$

Factoring By Grouping

Some quadrinomials can be factored using a method called **factoring by grouping**. This technique requires us to use parentheses to break the quadrinomial into a sum of two binomials, called *groups*. It is most common to group the first two terms separately from the last two terms.

Important Note: The two groups should always be separated by a **plus** sign.

For example, use factoring by grouping to factor the polynomial.

$5x^3 + 15x^2 + 2x + 6$	Create the sum of two groups; separate them with a plus sign.
$= (5x^3 + 15x^2) \;\mathbf{+}\; (2x + 6)$	Extract the GCF of each group separately.
$= 5x^2(\boldsymbol{x} + \mathbf{3}) \;\mathbf{+}\; 2(\boldsymbol{x} + \mathbf{3})$	Extract the common binomial factor, $(\boldsymbol{x} + \mathbf{3})$.
$= (5x^2 + 2)(\boldsymbol{x} + \mathbf{3})$	This could also be written as $(x + 3)(5x^2 + 2)$.

Group Exercise 5 Factor the quadrinomial using factoring by grouping.

a) $4x^3 + 6x^2 + 10x + 15$

b) $6y^3 - 8y^2 + 3y - 4$

If the third term in a quadrinomial is negative, we must still put a ***plus* sign** between the two groups. This creates a negative lead term in the second group, so we must factor out a negative GCF from the second group.

For example, use factoring by grouping to factor the polynomial.

	$14x^3 + 21x^2 - 4x - 6$	Create the sum of two groups; separate them with a plus sign.
$=$	$(14x^3 + 21x^2) + (-4x - 6)$	Extract the GCF of each group separately. In the second group, the GCF must be negative.
$=$	$7x^2\mathbf{(2x + 3)} + -2\mathbf{(2x + 3)}$	Extract the common binomial factor, $\mathbf{(2x + 3)}$.
$=$	$(7x^2 - 2)\mathbf{(2x + 3)}$	This could also be written as $(2x + 3)(7x^2 - 2)$.

Notice that the $+ -2$ is written as *minus* 2 in the first binomial factor.

Group Exercise 6 Factor the quadrinomial using factoring by grouping.

a) $10xy - 8x - 15y + 12$

b) $w^3 + 4w^2 - 5w - 20$

Focus Exercises

Factor out the GCF from each polynomial. Check all answers for accuracy.

1. $14x^2 - 7x$

2. $5x^2 + 4$

3. $16b^6 + 24b^5 - 8b^4$

Factor out the negative GCF from each polynomial. Check all answers for accuracy.

4. $-6x^3 - 18x$

5. $-m^3 - 6m$

6. $-12p^6 - 3p^4 + 6p^2$

Use factor by grouping to factor each quadrinomial. If a quadrinomial is not factorable, write prime.

7. $4w^3 + 3w^2 + 20w + 15$

8. $6xy - 8x + 3y - 4$

9. $6x^3 + 3x^2 - 2x - 1$

10. $m^3 + 6m^2 - 2m - 12$