# Factoring Trinomials

## FACTORING QUADRATICS

A **quadratic** polynomial is in the form $ax^2 + bx + c$, for which $a \neq 0$. However, $b$ or $c$ may be 0. If $b$ or $c$ is 0, then the quadratic has fewer than three terms. For example, these are all quadratic polynomials:

a) $2x^2 + 7x + 6$ b) $w^2 - 36$ c) $5m^2 + 10m$

**Group Exercise 1** Which of the following is a quadratic polynomial? If not, why not?

a) $2x^3 - 7x + 1$ b) $5w - 20$ c) $8y^2 + 9$ d) $3x + 7x^2$

Some **quadratic** trinomials can be factored into a product of two binomials. For example, each of these is factored as shown below. We can verify that the factoring is correct by multiplying the binomials (using the FOIL method or other technique.)

| a) | b) | c) |
| --- | --- | --- |
| $2x^2 + 7x + 6$ | $3w^2 - 20w + 12$ | $6m^2 + 7m - 5$ |
| factors into | factors into | factors into |
| $= (2x + 3)(x + 2)$ | $= (w - 6)(3w - 2)$ | $= (3m + 5)(2m - 1)$ |
| Verify using FOIL: | Verify using FOIL: | Verify using FOIL: |
| **F O I L** | **F O I L** | **F O I L** |
| $= 2x^2 + 4x + 3x + 6$ | $= 3w^2 - 2w - 18w + 12$ | $= 6m^2 - 3m + 10m - 5$ |
| $= \mathbf{2x^2 + 7x + 6}$ ✓ | $= \mathbf{3w^2 - 20w + 12}$ ✓ | $= \mathbf{6m^2 + 7m - 5}$ ✓ |

Each of these trinomials matches (✓) the trinomials we started with, so we have verified that the factoring is correct.

**Group Exercise 2** Verify that the factoring shown is the correct factoring by using FOIL.

a) $3x^2 + 8x + 4$

$= (3x + 2)(x + 2)$

b) $x^2 + 7x - 60$

$= (x + 12)(x - 5)$

The question is, how do we get to those binomial factors? Do we just guess? No. There is a process, and the more you use it, the better you will become at factoring trinomials.

First, factoring is the reverse process of multiplication, so we can think about reversing the steps of multiplying binomials (FOIL) and apply it to factoring trinomials.

For example, let's multiply $(3x + 2)(x - 6)$. We start by finding the four **FOIL** products:

$$(3x + 2)(x - 6)$$

| | | F O I L |
|---|---|---|
| The four **FOIL** products: | $=$ | $3x^2 \underline{- 18x + 2x} - 12$ |
| The two middle terms are like terms and can combine, giving us a trinomial: | $=$ | $3x^2 \underline{- 16x} - 12$ |

This means **(1)** $(3x + 2)(x - 6)$ multiplies out to $3x^2 - 16x - 12$ and **(2)** $3x^2 - 16x - 12$ factors into $(3x + 2)(x - 6)$

The reverse of this includes the following steps. Starting with a trinomial $3x^2 - 16x - 12$, write the middle term as two like terms. This will create a four-term polynomial which we can factor using factoring by grouping:

| | | |
|---|---|---|
| Start with a trinomial: | | $3x^2 \underline{- 16x} - 12$ |
| Split the middle term into two terms and write the trinomial with four terms: | $=$ | $3x^2 \underline{- 18x + 2x} - 12$ |
| Use factoring by grouping: | $=$ | $(3x^2 - 18x) + (2x - 12)$ |
| | $=$ | $3x(x - 6) + 2(x - 6)$ |
| and here is the factored form: | $=$ | $\mathbf{(3x + 2)(x - 6)}$ |

The most important question in all of this is:

★ How do we know what two like terms to write in place of the middle term? ★

In other words, how do we know to break up $\underline{-16x}$ into $\underline{-18x \text{ and } +2x}$?

The answer to this question is, the Factor Game.

## TRINOMIALS AND THE FACTOR GAME

Now let's put the Factor Game to good use. Consider a trinomial of the form $ax^2 + bx + c$, where $a$, $b$, and $c$ are integers, and $a > 0$.

> In the trinomial $ax^2 + bx + c$, the Product number is $\boldsymbol{a \cdot c}$, and the Sum number is $\boldsymbol{b}$.

**Example 2:** Given the trinomial, identify the Product and Sum numbers, and find the winning combination of the Factor Game, if there is one.

a) $3x^2 - 14x + 8$ b) $5y^2 - 4y - 12$ c) $3w^2 + 6w - 8$

**Procedure:** Identify the trinomial values of $a$, $b$, and $c$ and find the product and sum numbers of the Factor Game. **Note:** It is possible that a trinomial does not have a winning combination.

**Answer:**

| a) | b) | c) |
|---|---|---|
| Product $= 3(8) = 24$<br>Sum $= -14$ | Product $= 5(-12) = -60$<br>Sum $= -4$ | Product $= 3(-8) = -24$<br>Sum $= 6$ |
| Combination: **-12 and -2** | Combination: **6 and -10** | There is **no combination.** |

**Group Exercise 3** Given the trinomial, identify the Product and Sum numbers, and find the winning combination for the Factor Game, if there is one.

a) $4x^2 + 12x + 5$ Product =

Sum =

Combination: ______________

b) $5x^2 - 13x + 6$ Product =

Sum =

Combination: ______________

c) $6x^2 - 13x - 4$ Product =

Sum =

Combination: ______________

d) $3x^2 + 4x - 4$ Product =

Sum =

Combination: ______________

## FACTORING TRINOMIALS USING THE FACTOR GAME

The whole point of the Factor Game is to be able to rewrite a trinomial into four terms. It is the winning combination of the Factor Game that tells us how to split up the trinomial's middle term into two terms, thereby making it a four-term polynomial.

Consider factoring the trinomial $8x^2 + 10x - 3$. Here are the steps to factoring a trinomial using the Factor Game and factoring by grouping.

(1) Identify the **Product** and **Sum** numbers and find the combination to the Factor Game:

$8x^2 + 10x - 3$ Product = 8(-3) = -24
Sum = + 10 The combination is +12 and -2.

The two factors in the combination are the coefficients of the two new $x$-terms that make the trinomial into a four-term polynomial. In other words, the middles term, + 10$x$, can be replaced by $+ 12x - 2x$:

$$8x^2 \underline{+ 10x} - 3$$

(2) Write the trinomial with four terms: $= 8x^2 \underline{+ 12x - 2x} - 3$

(3) Use factoring by grouping: $= (8x^2 + 12x) + (-2x - 3)$

$$= 4x\mathbf{(2x + 3)} + -1\mathbf{(2x + 3)}$$

$$= (4x - 1)\mathbf{(2x + 3)}$$

We can verify that this factoring is true by multiplying the binomials together using **FOIL**.

If the Factor Game has no winning combination, then the trinomial is not factorable, and we say that it is *prime*.

**Example 3:** Factor each trinomial by using the Factor Game to rewrite it as a four-term polynomial. Then use factoring by grouping.

a) $6p^2 + 5p - 6$ b) $5x^2 - 16x + 12$ c) $3r^2 + 6r - 8$

**Procedure:** Identify the Product and Sum numbers and play the Factor Game. Use the winning combination to write the trinomial as a four-term polynomial, and use factoring by grouping. Verify that the result is true by using **FOIL** to multiply the results.

**Answer:**

a) $6p^2 + 5p - 6$ Product = 6(-6) = -36
Sum = + 5 The combination is -4 and -9.

$6p^2 + 5p - 6$ Split the middle term into $-4p + 9p$ :

$= 6p^2 - 4p + 9p - 6$ Now show the groupings:

$= (6p^2 - 4p) + (9p - 6)$ Factor out the common monomial factor from each group.

$= 2p(3p - 2) + 3(3p - 2)$ and factor out $(2p + 3)$:

$= \mathbf{(2p + 3)(3p - 2)}$ Now verify this factoring by multiplying the binomials.

b) $5x^2 - 16x + 12$

Product $= 5(12) = 60$
Sum $= -16$

The combination is -10 and -6.

$5x^2 - 16x + 12$ Split the middle term into $-10x - 6x$:

$= 5x^2 - 10x - 6x + 12$ Now show the groupings:

$= (5x^2 - 10x) + (-6x + 12)$ Factor out the common monomial factor from each group.

$= 5x(x - 2) + -6(x - 2)$ Now factor out $(x - 2)$:

$= \mathbf{(5x - 6)(x - 2)}$ Now verify this factoring by multiplying the binomials.

c) $3r^2 + 6r - 8$

Product $= 3(-8) = -24$
Sum $= +6$

There is **no combination**.

Because there is no combination to the Factor Game, the trinomial is ***prime***.

**Group Exercise 4** Factor each trinomial by using the Factor Game to rewrite it as a four-term polynomial. Then use factoring by grouping. If the trinomial cannot be factored, write "prime."

a) $3y^2 + 11y + 6$

b) $2x^2 - 5x + 4$

c) $m^2 + m - 30$

d) $6p^2 - p - 2$

## Focus Exercises

*Factor each trinomial by using the Factor Game. If the trinomial is not factorable, write "prime."*

**1.** $4x^2 + 11x + 6$

**2.** $2w^2 - 9w + 9$

**3.** $2m^2 + 6m + 9$

**4.** $p^2 - 12p + 36$

**5.** $10y^2 + 9y + 2$

**6.** $4x^2 - 12x + 5$

**7.** $5v^2 - 6v - 8$

**8.** $12k^2 - 8k + 1$

**9.** $h^2 + 16h + 64$

**10.** $10x^2 + 3x - 4$