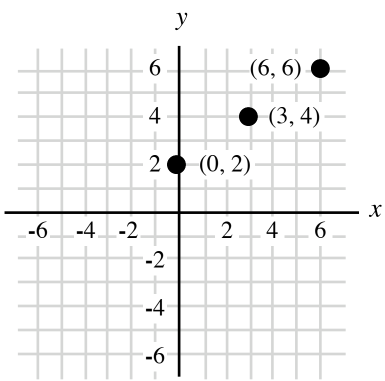
Introduction to Functions: Domain and Range

RELATIONS: DOMAIN AND RANGE

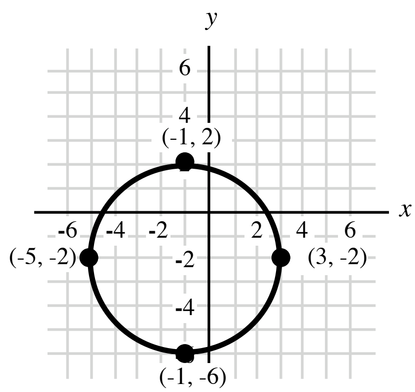
A **relation** is a set of ordered pairs. A relation can be a simple set of just a few ordered pairs, such as {(0, 2), (1, 3), (2, 4)}, or it can be infinite, such as the set of all points on a line or a curve.

The **domain** of a relation is the set of all (possible) *x*-values, and the **range** is the set of all *y*-values.



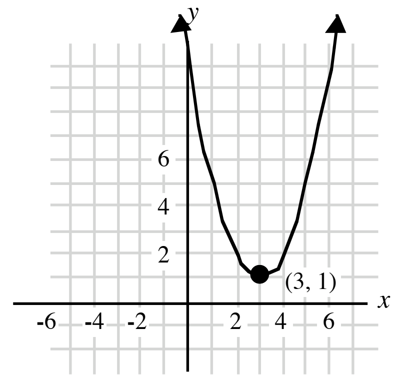
Domain: {0, 3, 6}

Range: {2, 4, 6}



Domain: [-5, 3]

Range: [-6, 2]



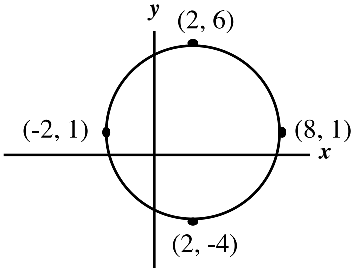
Domain: (-∞, ∞)*

Range: [1, ∞)

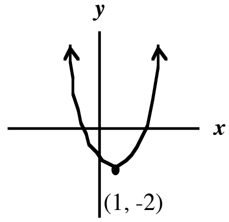
The domain and range are sets of numbers and we can represent each in one of several ways. In this appendix, you might come across any of these solution sets:

Words	Interval Notation	Set Builder Notation	Symbolically
all real numbers	$(-\infty, \infty)$	$\{ x \mid x$ is a real number $\}$	$\mathbb{R}$
x is between -5 and 3, inclusive	$[-5, 3]$	$\{ x \mid -5 \le x \le 3 \}$	$-5 \le x \le 3$
x is between -5 and 3, exclusive	$(-5, 3)$	$\{ x \mid -5 < x < 3 \}$	$-5 < x < 3$
x is greater than or equal to -1	$[-1, \infty)$	$\{ x \mid x \ge -1 \}$	$x \ge -1$
x is less than 2	$(-\infty, 2)$	$\{ x \mid x < 2 \}$	$x < 2$
x is not 7	$(-\infty, 7) \cup (7, \infty)$	$\{ x \mid x \ne 7 \}$	$\mathbb{R} - \{ 7 \}$

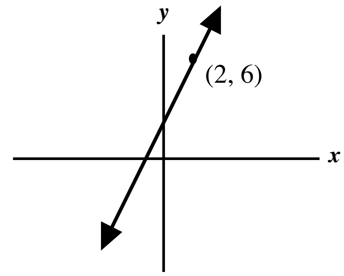
You Try It 1 For each relation, identify both the domain and the range.

<table>
<tr>
<td>a)


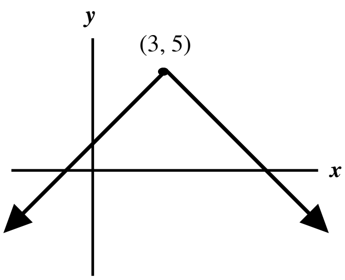
Domain:

Range:</td>
<td>b)


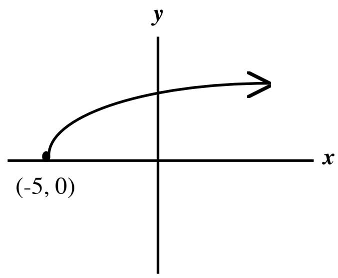
Domain:

Range:</td>
<td>c)


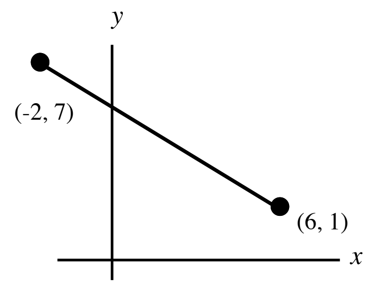
Domain:

Range:</td>
</tr>
<tr>
<td>d)


Domain:

Range:</td>
<td>e)


Domain:

Range:</td>
<td>f)


Domain:

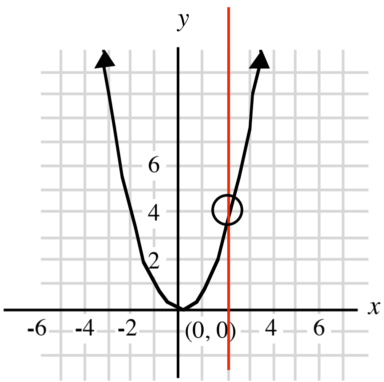
Range:</td>
</tr>
</table>

FUNCTION **D**EFINITION

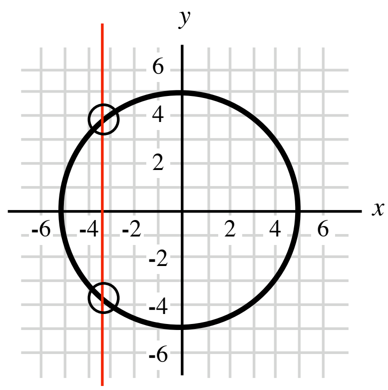
A **function** is a relation such that, for every *x* there is only one *y*.

The *vertical line test* is a way to visually identify whether the graph of a relation is a function. If any vertical line can cross the graph in more than one place, then the graph is *not* a function; otherwise, it is a function.

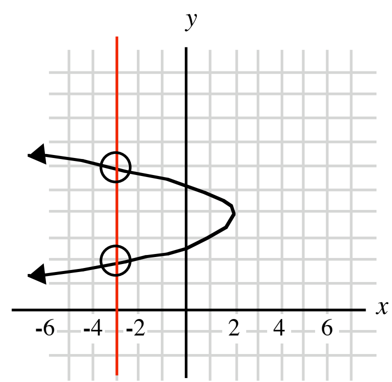
The basic idea behind the vertical line test is that we are visually checking each and every *x*-value in the domain, making sure that it corresponds to only one *y*-value.



This relation is a function.



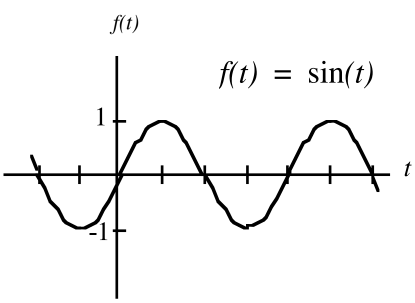
This relation is *not* a function.

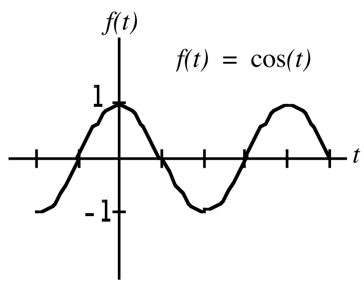


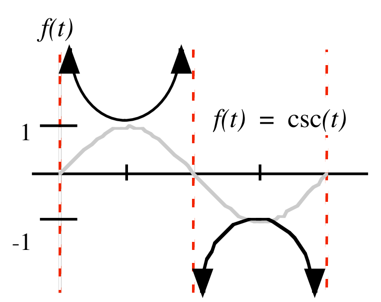
This relation is *not* a function.

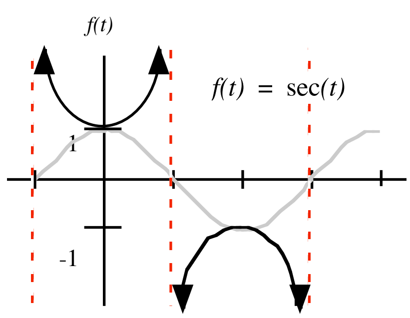
GRAPHS OF **T**RIGONOMETRIC **F**UNCTIONS

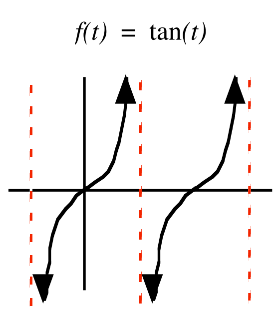
These are all functions:

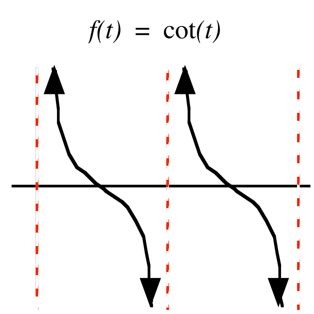












RESTRICTIONS ON THE DOMAIN

Some functions have natural restrictions. In particular,

(i) a denominator can never be 0 (The numerator is unaffected by this restriction.)

(ii) the radicand of a square root can never be negative. We do not consider the option of imaginary numbers because the x- and y-axes are real number axes.

Example 1: Identify the domain of each function.

a) $y = \sqrt{2x + 1}$ b) $y = \dfrac{x + 2}{3x - 4}$ c) $y = x^2 + 4x - 1$

Procedure: Identify whether the function has a natural domain restriction or otherwise.

Answer:

a) The radicand cannot be negative:

$$2x + 1 \geq 0$$

$$2x \geq -1$$

Domain: $x \geq -\frac{1}{2}$

b) The denominator cannot be zero:

$$3x - 4 \neq 0$$

$$3x \neq 4$$

Domain: $x \neq \frac{4}{3}$

c) For polynomial functions, the domain is <u>all real numbers</u> (unless it has a given domain restriction).

Domain: $\mathbb{R}$

Note: For some functions, the range is not intuitive and is often found only after the function has been graphed.

<u>You Try It 2</u> For each function, identify the domain. (You are not asked to find the range.)

a) $y = \dfrac{2x - 9}{3 - 6x}$ b) $y = -x^3 + x - 2$ c) $y = \sqrt{4 - 5x}$

You Try It Answers

YTI 1:

a) Domain: $-2 \le x \le 8$
Range: $-4 \le y \le 6$

b) Domain: $\mathbb{R}$
Range: $y \ge -2$

c) Domain: $\mathbb{R}$
Range: $\mathbb{R}$

d) Domain: $\mathbb{R}$
Range: $y \le 5$

e) Domain: $x \ge -5$
Range: $y \ge 0$

c) Domain: $-2 \le x \le 6$
Range: $1 \le y \le 7$

YTI 2:

a) Domain: $x \ne \frac{1}{2}$

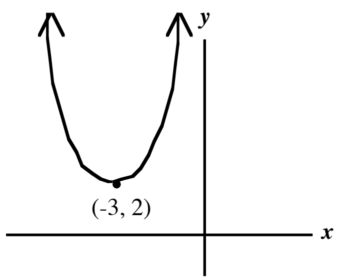
b) Domain: $\mathbb{R}$

c) Domain: $x \le \frac{4}{5}$

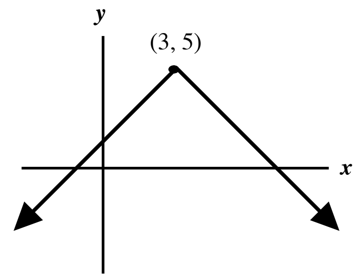
Focus Exercises

Given the graph of f(x), determine its domain and range.

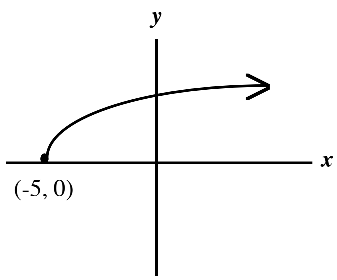
1.



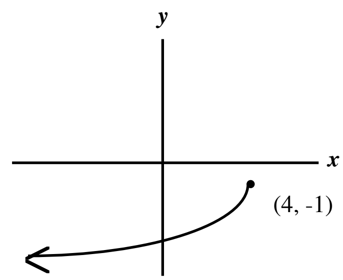
2.



3.



4.



Identify the domain of the function. Keep in mind any possible restriction the domain may have.

5. $f(x) = \sqrt{2x - 6}$

6. $h(x) = \dfrac{x + 1}{3x - 5}$

7. $g(x) = \dfrac{2}{3}x - 4$

8. $f(x) = \sqrt{8 - 4x}$

9. $k(x) = \dfrac{x}{x^2 - 4}$

10. $f(x) = x^2 + 1$