Graphing the Reciprocal Function

Division by 0

Concept	Example
We cannot divide by 0 (zero)	
a) $\frac{a}{0}$ is ***undefined***. for all values of a.	**a)** $\frac{4}{0}$ and $\frac{-5}{0}$ are both undefined.
b) If $f(x) = \frac{a}{x}$, then $f(x)$ is undefined when the value of x is 0. This means that 0 (zero) is not in the domain of $f(x)$.	**b)** If $f(x) = \frac{7}{x}$, then the domain of $f(x)$ is all real numbers except 0: Domain: $\mathcal{R} - \{0\}$.
c) If $g(x) = \frac{a}{binomial}$, then $g(x)$ is undefined when the binomial is 0, and that value is **excluded** from the domain of $g(x)$.	**c)** If $g(x) = \frac{-3}{x-6}$, then the domain of $g(x)$ is all real numbers except 6: Domain: $\mathcal{R} - \{6\}$.

Group Exercise 1 Identify the domain of the given function.

a) $f(x) = \frac{4}{x-5}$

b) $g(x) = \frac{-2}{x+4}$

c) $h(x) = \frac{-1}{2x-7}$

d) $k(x) = \frac{1}{3x+8}$

e) $f(x) = \frac{x+5}{4x-8}$

f) $g(x) = \frac{x-6}{2x+6}$

The Reciprocal Function

a) $f(x) = \frac{1}{x}$ is called the ***reciprocal function***.	**a)** $f\left(\frac{4}{7}\right) = \frac{7}{4}$, the reciprocal of $\frac{4}{7}$
b) The smaller the value of x, the larger the value of its reciprocal:	**b)** $f\left(\frac{1}{1{,}000}\right) = 1{,}000$
c) The larger the value of x, the smaller the value of its reciprocal:	**c)** $f(1{,}000) = \frac{1}{1{,}000}$
d) The reciprocal of a negative number is negative.	**d)** $f\left(\frac{-1}{3}\right) = -3$

Group Exercise 2 Given $f(x) = \frac{1}{x}$, find the value of each. Use the values to sketch the graph of $f(x) = \frac{1}{x}$.

a) $f(4) =$ b) $f(1) =$

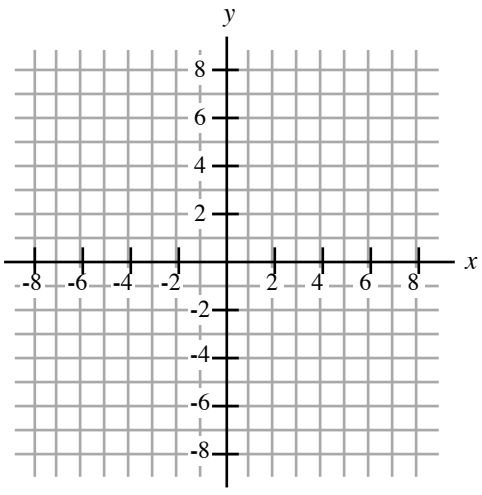
c) $f\left(\frac{1}{2}\right) =$ d) $f\left(\frac{1}{3}\right) =$

e) $f\left(\frac{1}{100}\right) =$ f) $f(8) =$

g) $f(-5) =$ h) $f(-1) =$

i) $f\left(-\frac{1}{2}\right) =$ j) $f\left(\frac{-1}{3}\right) =$

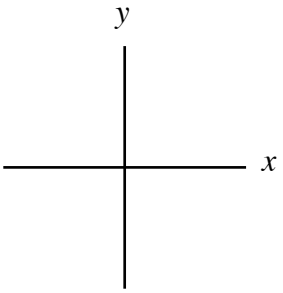
k) $f\left(\frac{-1}{100}\right) =$ l) $f(-6) =$



Group Exercise 3 Based on your graph of $f(x) = \frac{1}{x}$, what happens to the graph at $x = 0$? Explain your answer.

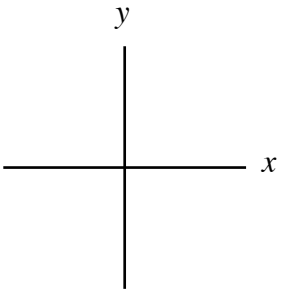
Group Exercise 4

Based on your graph of $f(x) = \frac{1}{x}$, draw a quick sketch of $f(x)$ in this x-y-plane.



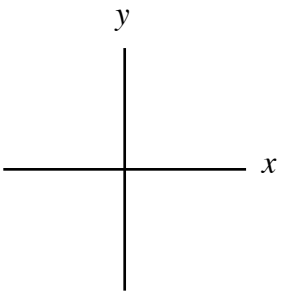
Group Exercise 5

Based on your quick sketch of $f(x) = \frac{1}{x}$ above, draw a quick sketch of $g(x) = \frac{2}{x}$ in this x-y-plane.



Group Exercise 6

Based on your quick sketch of $f(x) = \frac{1}{x}$ above, draw a quick sketch of $h(x) = \frac{-1}{x}$ in this x-y-plane.



Asymptotes

Based on the reciprocal function, $f(x) = \frac{1}{x}$, from the various graphs we have drawn, no points ever cross the y-axis. This is because $x = 0$ is not in the domain (so there can't be a y-value when $x = 0$).

Instead, we have an *asymptote* right along the y-axis. An ***asymptote*** is a line—usually horizontal or vertical—that acts as a guideline to a graph.

Group Exercise 7

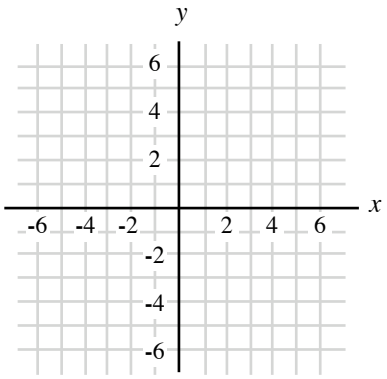
a) For the graph of $f(x) = \frac{1}{x}$, how is the y-axis an asymptote?

b) Is this a vertical asymptote or a horizontal asymptote?

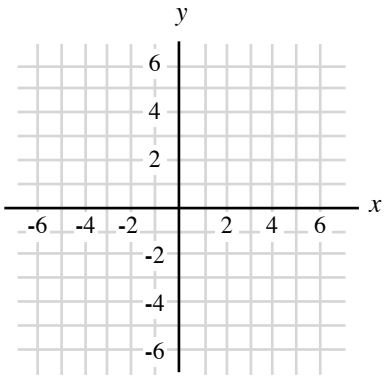
c) Does the graph have any other asymptotes? Explain.

Group Exercise 8 For each, identify the domain and draw a *quick sketch* of each of the following functions. Use a dotted line for any vertical asymptotes.

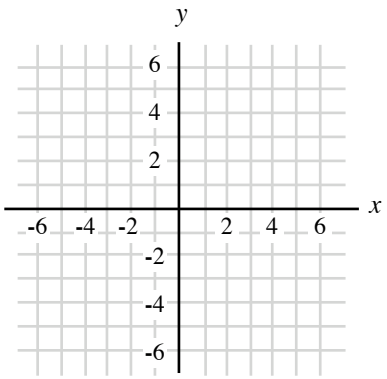
a) $f(x) = \dfrac{1}{x-3}$



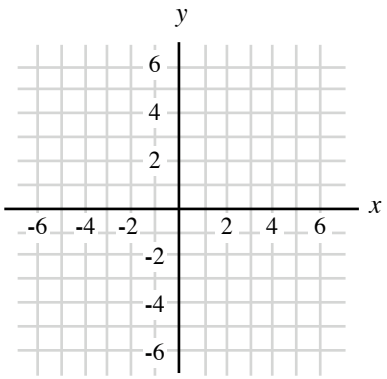
b) $f(x) = \dfrac{1}{x+4}$



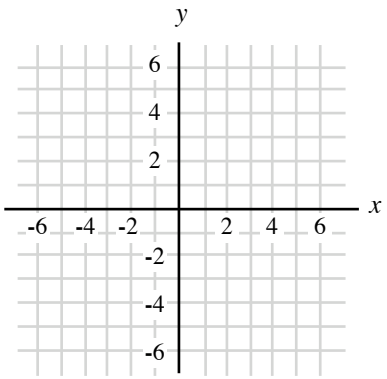
c) $f(x) = \dfrac{3}{x+1}$



d) $f(x) = \dfrac{2}{x-5}$



e) $f(x) = \dfrac{3}{x+1}$



f) $f(x) = \dfrac{2}{x-5}$