Multiplying and Dividing Radicals

MULTIPLICATION

Let's take a closer look at the second part of the Product Rule of Radicals:

The Product Rule of Radicals

As long as both $x \geq 0$ and $y \geq 0$, then

2. $\sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y}$

This tells us that the product of two radicals can be written as a single radical. We can see this work with the following examples:

Example A_1: $\sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$

Example A_2: $\sqrt{4 \cdot 9} = \sqrt{36} = 6$

Example 1: Multiply and simplify.

a) $\sqrt{3} \cdot \sqrt{5}$ b) $\sqrt{6} \cdot \sqrt{10}$ c) $(\sqrt{5})^2$

Procedure: Use the Product Rule of Radicals to multiply. Simplify the result, if possible.

Answer:

a) $\sqrt{3} \cdot \sqrt{5}$

$= \sqrt{3 \cdot 5}$

$= \sqrt{15}$ Cannot simplify.

b) $\sqrt{6} \cdot \sqrt{10}$

$= \sqrt{6 \cdot 10}$

$= \sqrt{60}$ Simplify.

$= \sqrt{4 \cdot 15}$

$= \sqrt{4} \cdot \sqrt{15}$

$= 2\sqrt{15}$

c) $(\sqrt{5})^2$

$= \sqrt{5} \cdot \sqrt{5}$

$= \sqrt{5 \cdot 5}$

$= \sqrt{25}$ Simplify.

$= 5$

This is just the original radicand.

Note: As demonstrated in Example 1c), the square of a square root radical is simply the whole number radicand. This means that no work is required to show that $(\sqrt{5})^2 = 5$.

You Try It 1 Multiply and simplify. Use Example 1 as a guide.

Perfect Squares
(You make the list.)

a) $\sqrt{15} \cdot \sqrt{2}$

b) $\sqrt{2} \cdot \sqrt{50}$

c) $\sqrt{2} \cdot \sqrt{10}$

d) $\sqrt{5} \cdot \sqrt{12}$

e) $(\sqrt{7})^2$

f) $(\sqrt{13})^2$

Consider the product $\sqrt{15} \cdot \sqrt{35}$. We can multiply directly to get $\sqrt{15 \cdot 35} = \sqrt{525}$. It might be difficult to tell, but $\sqrt{525}$ does simplify:

$$\sqrt{525} = \sqrt{25 \cdot 21} = \sqrt{25} \cdot \sqrt{21} = 5\sqrt{21}$$

Instead of multiplying directly, and creating a rather large radicand, we have another option. Before multiplying the radicals, we can find the prime factorization of each radical and then multiply:

$\sqrt{15} \cdot \sqrt{35}$	Write the prime factorization of each radicand.
$= \sqrt{3 \cdot 5} \cdot \sqrt{7 \cdot 5}$	Use the product rule to multiply the radicals in their prime factored form.
$= \sqrt{3 \cdot 5 \cdot 7 \cdot 5}$	Identify duplicate pairs of prime factors, $5 \cdot 5$, and reorder the prime factorization.
$= \sqrt{(5 \cdot 5) \cdot (3 \cdot 7)}$	Multiply within the groups.
$= \sqrt{25 \cdot 21}$	Separate the radicals and simplify.
$= \sqrt{25} \cdot \sqrt{21}$	
$= 5\sqrt{21}$	*This was more steps than is actually necessary, but it's best to show you all of the work than not enough.*

You Try It 2 Multiply and simplify. Use the discussion above as a guide.

a) $\sqrt{14} \cdot \sqrt{21}$

b) $\sqrt{6} \cdot \sqrt{50}$

Perfect Squares
(You make the list.)

c) $\sqrt{18} \cdot \sqrt{8}$

d) $\sqrt{12} \cdot \sqrt{75}$

THE QUOTIENT RULE OF RADICALS

The **Quotient Rule of Radicals** is similar to the Product Rule of Radicals; it uses division instead of multiplication::

The Quotient Rule of Radicals

1. $\dfrac{\sqrt{x}}{\sqrt{y}} = \sqrt{\dfrac{x}{y}}$ for $y \neq 0$

and

2. $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$ for $y \neq 0$

Part 1 of this quotient rule allows us to write a fraction of radicals as a single radical.

For example, $\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{\dfrac{18}{2}} = \sqrt{9} = 3$

and $\dfrac{\sqrt{20}}{\sqrt{2}} = \sqrt{\dfrac{20}{2}} = \sqrt{10}$

Example 2: Use part 1 of the Quotient Rule of Radicals to simplify the following expressions.

a) $\dfrac{\sqrt{20}}{\sqrt{5}}$ b) $\dfrac{\sqrt{18}}{\sqrt{3}}$ c) $\dfrac{\sqrt{60}}{\sqrt{5}}$

Procedure: Write each expression as a single fraction within a radical, then simplify, if possible.

Answer:

a) $\dfrac{\sqrt{20}}{\sqrt{5}} = \sqrt{\dfrac{20}{5}} = \sqrt{4} = 2$

b) $\dfrac{\sqrt{18}}{\sqrt{3}} = \sqrt{\dfrac{18}{3}} = \sqrt{6}$ which can't be simplified further

c) $\dfrac{\sqrt{60}}{\sqrt{5}} = \sqrt{\dfrac{60}{5}} = \sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

You Try It 3 Use part 1 of the Quotient Rule of Radicals to simplify the following expressions. Use Example 2 as a guide.

a) $\dfrac{\sqrt{45}}{\sqrt{5}}$

b) $\dfrac{\sqrt{75}}{\sqrt{3}}$

c) $\dfrac{\sqrt{21}}{\sqrt{3}}$

d) $\dfrac{\sqrt{56}}{\sqrt{7}}$

IF THE DENOMINATOR IS A PERFECT SQUARE ...

Part 2 of the quotient rule allows us to separate the square root of a fraction, such as $\sqrt{\dfrac{10}{4}}$, into two radicals, one divided by the other:

$$\sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{\sqrt{4}} = \frac{\sqrt{10}}{2}$$

Caution: $\dfrac{\sqrt{10}}{2}$ cannot simplify any further. The 10 and 2 cannot combine directly because 2 is not within a radical.

Example 3: Use part 2 of the Quotient Rule of Radicals to simplify each expression.

a) $\sqrt{\frac{25}{4}}$ b) $\sqrt{\frac{26}{9}}$ c) $\sqrt{\frac{30}{25}}$

Procedure: Separate each expression into two radicals. Simplify if possible.

Answer:

a) $\sqrt{\frac{25}{4}}$ b) $\sqrt{\frac{26}{9}}$ c) $\sqrt{\frac{30}{25}}$

$= \frac{\sqrt{25}}{\sqrt{4}}$ $= \frac{\sqrt{26}}{\sqrt{9}}$ $= \frac{\sqrt{30}}{\sqrt{25}}$

$= \frac{5}{2}$ $= \frac{\sqrt{26}}{3}$ $= \frac{\sqrt{30}}{5}$

You Try It 4 Use part 2 of the Quotient Rule of Radicals to simplify each expression. Use Example 3 as a guide.

a) $\sqrt{\frac{49}{9}}$ b) $\sqrt{\frac{16}{81}}$

c) $\sqrt{\frac{15}{64}}$ d) $\sqrt{\frac{21}{9}}$

You Try It Answers

You Try It 1	a)	$\sqrt{30}$	b)	10	c)	$2\sqrt{5}$	d)	$2\sqrt{15}$
	e)	7	f)	13				
You Try It 2	a)	$7\sqrt{6}$	b)	$10\sqrt{3}$	c)	12	d)	30
You Try It 3	a)	3	b)	5	c)	$\sqrt{7}$	d)	$2\sqrt{2}$
You Try It 4	a)	$\frac{7}{3}$	b)	$\frac{4}{9}$	c)	$\frac{\sqrt{15}}{8}$	d)	$\frac{\sqrt{21}}{3}$

Focus Exercises

Use the Product Rule of Radicals to write each as one radical. Simplify, if possible.

1. $\sqrt{5} \cdot \sqrt{3}$

2. $\sqrt{11} \cdot \sqrt{5}$

3. $\sqrt{18} \cdot \sqrt{2}$

4. $\sqrt{8} \cdot \sqrt{2}$

5. $(\sqrt{12})^2$

6. $(\sqrt{18})^2$

7. $\sqrt{20} \cdot \sqrt{2}$

8. $\sqrt{15} \cdot \sqrt{3}$

9. $\sqrt{7} \cdot \sqrt{14}$

10. $\sqrt{10} \cdot \sqrt{8}$

11. $\sqrt{6} \cdot \sqrt{42}$

12. $\sqrt{10} \cdot \sqrt{20}$

Use the Quotient Rules of Radicals to simplify the expression completely.

13. $\dfrac{\sqrt{90}}{\sqrt{10}}$

14. $\dfrac{\sqrt{75}}{\sqrt{3}}$

15. $\sqrt{\dfrac{1}{36}}$

16. $\sqrt{\dfrac{81}{100}}$

17. $\sqrt{\dfrac{35}{36}}$

18. $\sqrt{\dfrac{21}{25}}$

19. $\sqrt{\dfrac{45}{81}}$

20. $\sqrt{\dfrac{24}{100}}$