Multiplying Binomials

MULTIPLYING BINOMIALS, THE FOIL METHOD

1. A **binomial** is two terms, added (or subtracted) together. Examples of binomials are

a) $3x + 7$ b) $5w^2 + 1$ c) $4 - 3y$ d) $2x - 9y$

2. The **product** *(multiplication)* of two binomials is written with parentheses around each binomial. These are all products of two binomials:

a) $(3x + 7)(x - 4)$ b) $(5w^2 + 1)(2w + 3)$ c) $(4 - 3y)(5 - y)$ d) $(2x - 9y)(2x - 9y)$

3. To multiply two binomials, we **distribute** the terms of the first binomial to the terms of the second binomial, creating **four** products added (or subtracted) together. These four terms are called a **quadrinomial**. For example:

a) $(3x + 7)(x - 4)$

$= 3x^2 - 12x + 7x - 28$

1 2 3 4

b) $(5w^2 + 1)(2w + 3)$

$= 10w^3 + 15w^2 + 2w + 3$

1 2 3 4

c) $(4 - 3y)(5 - y)$

$= 20 - 4y - 15y + 3y^2$

1 2 3 4

d) $(2x + 9y)(2x - 9y)$

$= 2x^2 - 18xy + 18xy - 81y^2$

1 2 3 4

4. Often (but not always), two of those terms are **like terms** (have the same variables and exponents) and can be combined. Usually it is the middle two terms of the quadrinomial. When combined, there are often three terms remaining, a **trinomial**. Sometimes, the like terms are opposites and add to 0 (zero) and only two terms remain, a binomial:

a) $(3x + 7)(x - 4)$

$= 3x^2 \underline{- 12x + 7x} - 28$

$= 3x^2 \underline{- 5x} - 28$ (trinomial)

b) $(5w^2 + 1)(2w + 3)$

$= 10w^3 + 15w^2 + 2w + 3$

(no like terms; still a quadrinomial)

c) $(4 - 3y)(5 - y)$

$= 20 \underline{- 4y - 15y} + 3y^2$

$= 20 \underline{- 19y} + 3y^2$ (trinomial)

d) $(2x + 9y)(2x - 9y)$

$= 2x^2 \underline{- 18xy + 18xy} - 81y^2$

$= 2x^2 - 81y^2$ (binomial)

5. One technique for multiplying binomials is called the **FOIL method**. It is a form of *distribution* that gives names to the four products, as described below. When multiplying two binomials, such as $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{c} + \boldsymbol{d})$, there are four pairs of terms in the following positions:

Pair of terms	Position	Represented by	
• a and c	**First** terms in each binomial When we distribute, $a \cdot c$ is called the **F**irst product (or *product of the* ***Firsts***).	**F**	**F**: $(\boldsymbol{a} + b)(\boldsymbol{c} + d)$
• a and d	the outermost, or **Outer**, terms in the entire product When we distribute, $a \cdot d$ is called the **O**uter product.	**O**	**O**: $(\boldsymbol{a} + b)(c + \boldsymbol{d})$
• b and c	the innermost, or **Inner**, terms in the entire product When we distribute, $b \cdot c$ is called the **I**nner product.	**I**	**I**: $(a + \boldsymbol{b})(\boldsymbol{c} + d)$
• b and d	the **Last** terms in each binomial When we distribute, $b \cdot d$ is called the **L**ast product.	**L**	**L**: $(a + \boldsymbol{b})(c + \boldsymbol{d})$

For example, in multiplying $(2x + 3)(4x - 5)$, we get four initial products. Based on the **FOIL** method, those products are:

F	O	I	L
First Product	Outer Product	Inner Product	Last Product
$2x \cdot 4x = \mathbf{8x^2}$	$2x \cdot (-5) = \mathbf{10x}$	$3 \cdot 4x = \mathbf{12x}$	$3 \cdot (-5) = \mathbf{-15}$

$$= 8x^2 \quad - \quad 10x \quad + \quad 12x \quad - \quad 15$$

$$= \mathbf{8x^2 + 2x - 15}$$

Group Exercise 1 Use the FOIL method to multiply these binomial products. Simplify if possible.

a) $(v + 5)(3v + 2)$

b) $(2y^2 - 3)(6y - 5)$

c) $(x - 5y)(3x + 4y)$

d) $(4 + 5x)(4 - 5x)$

6. If the **O**uter and **I**nner products are like terms, then they can be combined, making the **FOIL** method more like **F + (O + I) + L**. When the sum of **O** and **I** are done mentally, this becomes a one-step process.

For example, to multiply $(3x + 1)(7x + 4)$, we can think of it as

F + (O + I) + L

$= 21x^2 + (12x + 7x) + 4$ ← This step can be done mentally.

$= 21x^2 + 19x + 4$

Group Exercise 2 Practice using the **FOIL** method along with the one-step technique to multiply and simplify.

a) $(2k + 5)(k - 6)$

b) $(2x + 3)(4x + 1)$

c) $(3 + 4x)(3 + 4x)$

d) $(5c + 1)(5c - 1)$

SQUARING A BINOMIAL

7. To square a binomial, such as $(2x + 5)^2$, one option is to write it as the product of the same binomial:

$(2x + 5)^2 = (2x + 5)(2x + 5)$. Using the **FOIL** method, the product is

F O I L

$= 4x^2 \ \underline{+\ 10x \quad +\ 10x} \ + 25$

$= 4x^2 \ \mathbf{+ 20x} + 25$

Because the Outer and Inner products are identical, their sum is just twice the Outer product:

F 2 · O L

$= 4x^2 + 2 \cdot (10x) + 25$

$= 4x^2 \ \mathbf{+ 20x} + 25$

In general, the square of a binomial fits one of these patterns:

The Square of a Binomial

1. $(a + b)^2 = a^2 + 2ab + b^2$ **2.** $(a - b)^2 = a^2 - 2ab + b^2$

Example 1: Multiply these squared binomials.

a) $(x + 7)^2$ b) $(3 - 4x)^2$ c) $(2x^2 + 5)^2$

Procedure: Multiply directly using **F + 2 · O + L**

Answer: a) $x^2 + 14x + 49$ b) $9 - 24x + 16x^2$ c) $4x^4 + 20x^2 + 25$

Caution: One common mistake in using this one-step technique is forgetting to double the Outer product, which is the middle term in the resulting trinomial.

Another common mistake is to square the first and last terms but forget to find the Outer product. This incorrectly leads to the resulting product having no middle term.

Group Exercise 3 Multiply these perfect square binomials and simplify.

a) $(x + 8)^2$ b) $(4w - 9)^2$

c) $(5 + 4y)^2$ d) $(6x - 2y)^2$

CONJUGATES

8. Two binomials are **conjugates** if their *first terms are exactly the same* but their <u>second terms are opposites</u>, as in $(a + b)$ and $(a - b)$. Conjugates always come in pairs. For example, $(x + 3)$ is not a conjugate without $(x - 3)$.

a) $(3y - 5)$ and $(3y + 5)$ are conjugates b) $(2 + 7x)$ and $(2 - 7x)$ are conjugates c) $(6x - 5y)$ and $(6x + 5y)$ are conjugates

9. When multiplying conjugates using the **FOIL** method, the **O**uter and **I**nner terms are always opposites and combine to 0, so <u>the product of a pair of conjugates is always the difference of squares</u>:

$$(a + b)(a - b) = a^2 - b^2$$

For example,

a) $(x - 3)(x + 3)$

$= x^2 + 3x - 3x - 9$ ← This step may be done mentally →

$= x^2 + 0x - 9$ ← This step may be done mentally →

$= x^2 - 9$

b) $(2 + 5x)(2 - 5x)$

$= 4 - 10x + 10x - 25x^2$ ← This step may be done mentally →

$= 4 + 0x - 25x^2$ ← This step may be done mentally →

$= 4 - 25x^2$

Example 2: Identify the conjugate of the given binomial and then multiply the pair of conjugates.

a) $(2r - 3)$ b) $(4x + y)$

Procedure: The conjugate of the given binomial will have the same first term but the opposite second term. The product of a pair of conjugates is always the difference of squares.

Answer:

a) Conjugate is $(2r + 3)$

$(2r - 3)(2r + 3)$

$a^2 - b^2$ ← This step may be done mentally →

$= (2r)^2 - (3)^2$ ← This step may be done mentally →

$= 4r^2 - 9$

b) Conjugate is $(4x - y)$

$(4x - y)(4x + y)$

$a^2 - b^2$

$= (4x)^2 - (y)^2$

$= 16x^2 - y^2$

<u>Group Exercise 3</u> Identify the conjugate of the given binomial and then multiply the pair of conjugates.

a) $(7x - 8)$ b) $(4w + 9)$

c) $(10m + p)$ d) $(1 - 6x)$

Focus Exercises

Multiply and simplify.

1. $(2x - 3)(3x + 8)$

2. $(5y - 6)(3y - 4)$

3. $(3 + 4u)(4 - 3u)$

4. $(7m - 10p)(5m + 3p)$

5. $(x + 7)^2$

6. $(1 - 8w)^2$

7. $(5y - 3)^2$

8. $(3r + 4p)^2$

Identify the conjugate of the given binomial and then multiply the pair of conjugates.

9. $(x + 5)$

10. $(6 - y)$

11. $(3r - 8)$

12. $(9p + 10)$