Simplifying Radicals

THE RADICAND

We know that 5^2 (5 squared) is 25, so 25 is called a perfect square number, or just *perfect square*. We also know that a square root of 25 is 5, written as $\sqrt{25} = 5$.

> The square root symbol, $\sqrt{}$, is called a **radical**, and the number within a radical is called the **radicand**.

So, in $\sqrt{25}$, the radicand is 25.

Whenever the radicand is a perfect square, such as 49, 16, 100, or 1, the resulting value is a whole number:

$$\sqrt{49} = 7, \quad \sqrt{16} = 4, \quad \sqrt{100} = 10, \quad \text{and} \quad \sqrt{1} = 1.$$

Many radicands are not prefect squares and their square root is an irrational number. For example,

$\sqrt{12} \approx 3.4641$. When we square 3.4641 we get a number very close to 12, but not exactly 12:

$$(3.4641)^2 = 11.99998881$$

In other words, there is no exact value for $\sqrt{12}$. Instead, we want to simplify $\sqrt{12}$ as best possible. This technique requires the **Product Rule of Radicals**.

SIMPLIFYING RADICALS

> **The Product Rule of Radicals**
>
> *As long as both $x \geq 0$ and $y \geq 0$, then*
>
> 1. $\sqrt{x \cdot y} = \sqrt{x} \cdot \sqrt{y}$
> 2. $\sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y}$

Part 1 of the Product Rule of Radicals is used to simplify radicals by <u>extracting a prefect square factor</u> from the radicand—if there is any. For example,

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2 \cdot \sqrt{3} \quad \text{or just} \quad 2\sqrt{3}$$

In this case, $2\sqrt{3}$, 2 is the *integer coefficient*.

Note: we cannot simplify a square root if it has no is a perfect square factors.

A radical expression might already have an integer coefficient before it is simplified; that coefficient is included in the final product. For example,

$$5\sqrt{12} = 5\sqrt{4 \cdot 3} = 5 \cdot \sqrt{4} \cdot \sqrt{3} = 5 \cdot 2 \cdot \sqrt{3} = 10\sqrt{3}$$

Example 1: Simplify each square root radical, if possible.

a) $\sqrt{24}$ b) $\sqrt{45}$ c) $7\sqrt{50}$ d) $\sqrt{30}$

Procedure: All of these can be factored in more than one way, but only one factorization will lead to a simplified radical. Look for perfect square factors for each.

Answer:

a) $\sqrt{24}$

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$ or just $2\sqrt{6}$

b) $\sqrt{45}$

$= \sqrt{9 \cdot 5}$

$= \sqrt{9} \cdot \sqrt{5}$

$= 3\sqrt{5}$

c) $7\sqrt{50}$

$= 7 \cdot \sqrt{25 \cdot 2}$

$= 7 \cdot \sqrt{25} \cdot \sqrt{2}$

$= 7 \cdot 5\sqrt{2}$

$= 35\sqrt{2}$

d) $\sqrt{30}$ *cannot be simplified* because 30 has no perfect square factors.

You Try It 1 Simplify the following, if possible. If the radicand has no perfect square factor, write *cannot be simplified*. (For your assistance, a list of perfect squares is shown at the right.) Use Example 1 as a guide.

a) $\sqrt{27}$

b) $\sqrt{28}$

c) $\sqrt{42}$

d) $\sqrt{90}$

e) $5\sqrt{18}$

f) $\frac{1}{2}\sqrt{20}$

Perfect Squares:

1
4
9
16
25
36
49
64
81
100

Sometimes, a radical can simplify more than just once. For example, $\sqrt{72}$ can first use $9\cdot 8$ to simplify, but the radicand can be simplified further:

$$\sqrt{72} = \sqrt{9\cdot 8} = 3\sqrt{8} = 3\sqrt{4\cdot 2} = 3\cdot 2\cdot\sqrt{2} = 6\sqrt{2}$$

You Try It 2 Simplify each radical expression. Use the discussion above as a guide.

a) $\sqrt{32}$

b) $\sqrt{162}$

Perfect Squares
(You make the list.)

c) $\sqrt{80}$

d) $\sqrt{360}$

You Try It Answers

You Try It 1	a)	$3\sqrt{3}$	b)	$2\sqrt{7}$	c)	cannot be simplified	d)	$3\sqrt{10}$
	e)	$15\sqrt{2}$	f)	$\sqrt{5}$				
You Try It 2	a)	$4\sqrt{2}$	b)	$9\sqrt{2}$	c)	$4\sqrt{5}$	d)	$6\sqrt{10}$

Focus Exercises

Simplify the following square roots, if possible. If the radicand has no perfect square factor, then write "cannot be simplified."

1. $\sqrt{45}$

2. $\sqrt{63}$

3. $\sqrt{40}$

4. $\sqrt{54}$

5. $\sqrt{200}$

6. $\sqrt{490}$

7. $10\sqrt{8}$

8. $7\sqrt{12}$

9. $4\sqrt{18}$

10. $2\sqrt{50}$

11. $5\sqrt{300}$

12. $3\sqrt{500}$