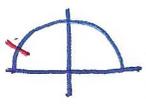


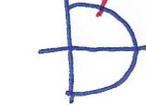
10. Evaluate each.

a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ 

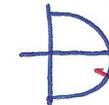
b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ 

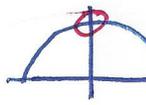
c) $\cos^{-1}(1) = 0$ 

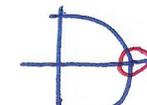
d) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ 

e) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ 

f) $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$ 

g) $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$ 

i) $\cos^{-1}(0) = \frac{\pi}{2}$ 

j) $\sin^{-1}(0) = 0$ 

11. Using the sum, difference, double angle, or half angle formulas, evaluate the following.

a) $\sin(15^\circ)$
There are a few options to make 15°: 60°-45° and 45°-30°. Either set is okay.
 $= \sin(45^\circ - 30^\circ)$
 $= \sin 45^\circ \cdot \cos 30^\circ - \sin 30^\circ \cdot \cos 45^\circ$
 $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$
 $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$

b) $2 \sin\left(\frac{13\pi}{12}\right) \cos\left(\frac{13\pi}{12}\right)$
 $= \sin(2A)$
 $= \sin\left(2 \cdot \frac{13\pi}{12}\right)$
 $= \sin\left(\frac{13\pi}{6}\right)$
 $= \frac{1}{2}$
This fits the pattern of $\sin(2A)$. In this case, $A = \frac{13\pi}{12}$

12. Find both $\sin(2A)$ and $\cos(2A)$ based on the given information. Simplify.

Q I
 $\sec A = \frac{3}{2}$, $0^\circ < A < 90^\circ$,
 Show work here:
 $\frac{x}{r} = \frac{2}{3} \rightarrow x = 2$
 now find y:
 $x^2 + y^2 = r^2$
 $(2)^2 + y^2 = (3)^2$
 $4 + y^2 = 9$
 $y^2 = 5$
 $y = \pm\sqrt{5}$
 $r = 3$
 $\cos A = \frac{2}{3}$
 $\sin A = \frac{y}{r} = \frac{\sqrt{5}}{3}$

This means $\frac{x}{r} = \frac{2}{3}$

a) $\sin(2A) = 2 \cdot \sin A \cdot \cos A$
 $= 2 \cdot \frac{\sqrt{5}}{3} \cdot \frac{2}{3}$
 $= \frac{4\sqrt{5}}{9}$

b) $\cos(2A) = \cos^2 A - \sin^2 A$
 $= \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$
 $= \frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

there are two other identities for $\cos(2A)$ that could be used.

Note: in part b), I use the technique of moving the denominator to the numerator before changing everything to sine/cosine.

This technique is optional.

now write everything in sin/cos.

13. Prove each identity.

a) $\csc x - \cot x \cos x = \sin x$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} \cdot \cos x =$$

$$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} =$$

$$\frac{1 - \cos^2 x}{\sin x} =$$

$$\frac{\sin^2 x}{\sin x} =$$

$$\sin x = \sin x$$

Q.E.D. ✓

b) $\frac{\csc x}{\cot x} - \frac{\cot x}{\csc x} = \sin x \tan x$

$$\csc x \cdot \tan x - \cot x \cdot \sin x =$$

$$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} =$$

$$\frac{1}{\cos x} - \cos x \cdot \frac{\cos x}{\cos x} =$$

$$\frac{1 - \cos^2 x}{\cos x} =$$

$$\frac{\sin^2 x}{\cos x} =$$

$$\sin x \cdot \frac{\sin x}{\cos x} =$$

$$\sin x \tan x = \sin x \tan x$$

Q.E.D. ✓

14. Solve for θ , $0^\circ \leq \theta < 360^\circ$.

$$\tan^2 \theta - \tan \theta = 0$$

$$\tan \theta (\tan \theta - 1) = 0$$

$$\tan \theta = 0 \text{ or } \tan \theta - 1 = 0$$

$$\tan \theta = 1$$

$$\theta = 0^\circ, 180^\circ \quad \theta = 45^\circ, 225^\circ$$

$$\theta = 0^\circ, 180^\circ, 45^\circ, 225^\circ$$

15. Solve for x , $0 \leq x < 2\pi$.

$$\sin 2x - \sin x = 0$$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0$$

$$\sin x = 0 \text{ or } 2 \cos x - 1 = 0$$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = 0, \pi$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$